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On the Diurnal Variations of the Earth's Magnetism Produced by the Moon and Sun

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VII. *On the Diurnal Variations of the Earth's Magnetism produced by the Moon and Sun.*

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Introduction.

§ 1. WHILE the observational study of terrestrial magnetism is receiving ever more and more attention, and being rewarded with success by the acquisition of new and important data, the theoretical side of the subject shows a much less rapid advance. The search for a physical theory of the earth's magnetism and its changes is fascinating but elusive. Perhaps in one case only—that of SCHUSTER's important theory* of the diurnal variations of the magnetic state of the earth—has there been put forward a clearly outlined theory which promises to explain the real mechanism of any magnetic phenomenon.

On this theory, the solar diurnal variations are attributed to the action of electromotive forces produced in masses of conducting air in the upper atmosphere, by their motion across the permanent magnetic field of the earth. The magnetic field of the resulting electric currents is identified with that which produces the observed diurnal changes. SCHUSTER has shown that if the motion of the air is taken to be substantially that which is indicated by the barometric variations, the atmosphere being supposed to oscillate as a whole, the conductivity required by the theory is not unreasonable, considering the ionization of the tenuous upper atmosphere by ultra-

* 'Phil. Trans.,' A, vol. 208, p. 163.

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violet radiation from the sun.* The fundamental assumptions are in accordance with SCHUSTER'S demonstration† that the magnetic variations are principally due to a system of currents above the earth's surface. In order to explain the relative magnitudes of the diurnal and semi-diurnal terms in the magnetic potential, it is necessary to suppose that the conductivity of the atmosphere varies with the solar hour angle, which is certainly *à priori* probable: the great excess of the summer variation over the winter variation is unexplained, however, as the usual rapid rate of recombination of ions makes it difficult to believe that the solar ionization is slowly cumulative.

There is at present much uncertainty as to the numerical constants of the potential of the magnetic field responsible for the solar diurnal variations, as the only two calculations yet made‡ show serious disagreement. A new determination of this potential is now in progress at the Royal Observatory, Greenwich. Whatever be the result of this calculation, however, there will remain several important features of the phenomenon which require explanation—in particular, the seasonal changes. By the elucidation of these difficulties, terrestrial magnetism may throw light on the ionization of the upper atmosphere. The variables at disposal in the theory are, unluckily, too numerous to get very definite knowledge of any one of them from a single source, and therefore it is peculiarly fortunate that there is a kindred but independent set of phenomena, produced by the moon jointly with the sun, which promises very valuably to supplement the knowledge furnished by the solar diurnal variations. It should be specially instructive to compare the seasonal changes of the two sets of phenomena.

§ 2. The general outlines of this paper may be briefly indicated here. The principal known facts regarding the lunar magnetic variation are first summarized, and it is shown that, so far as they go, they seem most easily explicable in the manner proposed by SCHUSTER for the solar diurnal variations. Nothing in the nature of a proof is yet possible however. Some new facts, deduced by harmonic analysis of existing material for the lunar variation at the separate phases of the moon, are then described, and it is pointed out how they confirm the hypothesis of the variable conductivity of the atmosphere in a very direct way, and provide a powerful means of quantitatively investigating the changes of the conductivity. The details of the calculation of these new harmonic terms in the lunar variation, and the actual tables of results, are collected in Part III. of the paper. In order to discuss the bearing of these observational results on the theory, it is necessary to extend SCHUSTER'S calculation of the effect of an atmospheric oscillation, under the influence of the earth's radial magnetic forces and the variable conductivity of the air, in producing

* That there is a highly conducting layer in the upper atmosphere is also indicated by the bending of electric waves round the earth.

† 'Phil. Trans.,' A, vol. 180, p. 467.

‡ SCHUSTER, 'Phil. Trans.,' A, vol. 180, p. 467; and FRITSCHE, St. Petersburg, 1902.

magnetic diurnal variations. The calculations are given in Part II., in a very general form; the work is in some respects simpler and more direct than in SCHUSTER's investigation, owing to the adoption of the resistivity, instead of the conductivity, as the variable. The formal results (which as yet, however, are at a somewhat incomplete stage) are reduced to numerical form and compared with the observed data. The whole of the discussion is collected in Part I., and it is shown that the fourth harmonic component of the lunar variation favours the assumption that the atmospheric conductivity may fall to a very small value during the night hours. The question of the seasonal variations, as affecting both the solar and lunar effects, is barely touched on, since though it arises naturally from the calculations in Part II., better observational material is necessary to realize the proper use of the theoretical work. A fuller discussion is reserved therefore till the new determination of the potential of the solar variation, already mentioned, is completed.

PART I.—*General Discussion.*

§ 3. The magnetic elements show regular periodic changes depending on the lunar hour angle, just as on the solar hour angle: the latter variations are considerably the greater of the two, and almost entirely mask the lunar variations. KREIL,* of Prague, in 1841, first established the existence of these changes, and since then a very limited number of investigators† have confirmed and extended KREIL's discovery. Owing to the nearly equal length of the solar and lunar days, the separation of the two effects involves considerable rearrangement of the observed data as usually tabulated, and the smallness of the lunar variation renders it necessary to deal with a large quantity of material in order to eliminate accidental errors. The determination of the lunar diurnal variation for the three magnetic elements at a single station is therefore a laborious undertaking, and hardly any observatory, as yet, includes such an examination of its observations in its scheme of work. If the potential of the magnetic field producing these variations is to be found, however, they must be computed not merely for one, but for several stations, well distributed on the earth's surface. This formidable task would be much

* Bohemian Society of Sciences, 1841.

† BROUN, 'Trevandrum Observations,' I., 1874; CHAMBERS, 'Phil. Trans.,' A, vol. 178, p. 1 (1887); 'Batavian Observations,' BERGSMA and VAN DER STOK, vols. I., III., IX., X., XVI., also 'Proc. Roy. Acad.,' Amsterdam, IV., 1887, and 'Archives Néerlandaises,' XVI.; FIGEE, 'Batavian Observations,' XXVI., 1903; LAMONT, 'Sitz. d. K. Akad. d. Wiss.,' 1864, t. 11, 2, Munich. SABINE, 'Phil. Trans.,' 1853, 1856, 1857; 'Roy. Soc. Proc.,' X., 1859-1860.

Also the published observations at St. Helena, Toronto, Hobarton, and Cape of Good Hope (edited by SABINE), and at Melbourne, Dublin, and Philadelphia. Also AIRY, 'Greenwich Observations,' 1859 and 1867.

Also MOOS, 'Bombay Magnetic Observations,' 1846-1905, vol. II. (1910); and VAN BEMMELEN, 'Met. Zeitschr.,' May, 1912.

expedited if various observatories would undertake the reduction of their own data on a uniform plan, and it is partly in the hope that some may be induced to co-operate in this work that the present preliminary paper has been written.

§ 4. When determined from the mean of a number of whole lunations, the lunar diurnal variation is found to be always of the same character, for every element and at every station: it consists solely of a very regular semi-diurnal oscillation. Other harmonic components of relatively small amplitude may be present, but their lack of regularity and consistency proves them to be accidental inequalities which are no real part of the phenomenon. This simplicity makes it probable that the lunar diurnal variation will be easier to explain than the solar diurnal variation.

SCHUSTER'S theory of the latter naturally suggests that the former is due to the lunar tidal oscillations of the atmosphere. These oscillations have very little effect upon the barometer, the ordinary diurnal barometric variation being a thermal and not a tidal effect; but a lunar barometric tide does exist, and has been evaluated with a considerable degree of accuracy at some tropical stations (St. Helena, Singapore, and Batavia).* The explanation gains weight from the fact that at perigee the lunar magnetic variations are of distinctly greater amplitude than at apogee,† and there is some evidence that the ratio of the amplitudes at the two seasons is that which would be predicted by the tidal theory (1·23), though the observational results do not suffice, as yet, to establish this definitely.

§ 5. Dr. VAN BEMMELEN, at Batavia, has recently collected all the existing determinations of the lunar magnetic variation for different stations, and has examined this material, together with newly computed data for other stations, to see whether the magnetic field which produces these effects has a potential, and whether the latter has its source above or below the earth's surface.‡ He finds that most of the field, at any rate, has a potential, and that this arises partly above and partly below the earth's surface, but that the internal field is too great to be merely a secondary induction effect. This result should be accepted with some reserve, at present, not only on account of the imperfections of the data, but also because the seasonal change of the variations was disregarded; in certain elements at some stations the summer and winter variations are of opposite sign, and this renders it unsafe to take the mean variation for the whole year. At many stations, unfortunately, the data so far computed apply only to the whole year, so that if this material was to be used, no course was possible save to adopt the mean of the year for all. One important result of VAN BEMMELEN'S work was to show that the principal term in the potential of the lunar variation field was of the form Q_3^2 (in the usual language of harmonic analysis, a tesseral harmonic of the second kind and third

* SABINE, 'Phil. Trans.,' 1847; 'Batavian Observations,' 28 (1905).

† See 'Trevandrum Observations' (BROUN), vol. I., p. 137, and SABINE'S and FIGEE'S discussions already cited.

‡ 'Met. Zeitschr.,' May, 1912.

degree). This is in accordance with the theory that the lunar atmospheric tide is the main cause of the phenomenon, although, of course, it does not prove this to be the case.

§ 6. So far reference has been made entirely to the lunar variation as determined from a number of whole lunations, as has been generally done (the exceptions are Trevandrum, Bombay, and Batavia). It will be remembered that SCHUSTER'S theory of the solar diurnal variation involved the hypothesis of a variable conductivity depending on the sun's hour angle. This should, of course, also affect the electric currents which arise from the lunar atmospheric tide, and so make the lunar magnetic variations depend on the sun as well as on the moon. In the course of a lunation, however, the angle between the sun and moon, viewed from the earth, changes from 0 to 2π, and the mean lunar variation for such a period cannot be expected to show any special dependence on solar time. At any particular lunar phase, however, the solar day hours, during which (over a given part of the earth) the atmospheric conductivity is greatest, occur at a definite part of the lunar day, this part changing with the lunar phase; and it has, in fact, been found* that the lunar variation determined from the mean of a number of days all at the same lunar phase is not of the semi-diurnal form. The variation curve goes through a regular cycle of change with lunar phase, in such a manner as to leave the mean variation over a whole lunation of the simple form already described. The magnetic needle is most mobile during the day hours: at certain seasons of the year, BROUN found that the amplitude of the lunar diurnal variation of magnetic declination at Trevandrum was five times as great during the solar day hours as during the night hours.† These facts clearly show that the conductivity of the medium in which the electric currents flow to produce the lunar magnetic variation depends on the position of the sun; and since it is unreasonable to suppose that the mechanisms concerned in producing the lunar and solar diurnal magnetic variations are materially different, the assumption of variable conductivity in SCHUSTER'S theory is confirmed in a very definite and independent way; in SCHUSTER'S discussion two barometric oscillations, diurnal and semi-diurnal, were concerned, and it was necessary to explain why the resulting magnetic variations, deduced on the assumption of uniform conductivity, did not bear the proper ratio to one another. This might be because the conductivity was not uniform, or because the ratio of the two oscillations was different in the upper regions of the atmosphere from that indicated by the barometer. This latter uncertainty is absent in the case of the lunar variations, where there is only a single barometric oscillation, from which arise magnetic variations of other periods, depending on the solar hour angle.

§ 7. In order to examine the effect of this variable conductivity, it is natural to determine the harmonic components of the lunar diurnal variation for different lunar

* By BROUN, CHAMBERS, FIGEE, and MOOS in the investigations already cited.

† 'Trevandrum Observations,' vol. I., p. 121.

phases, but (rather strangely) this has only once been done hitherto, and then without result.* CHAMBERS† obtained an analytical expression for the variation and its dependence on phase, which satisfactorily represents the observations, but it is not of a simple character. His formula was

$$f_{c,2}(h)\cos 2\left(\frac{2\pi}{P}t\right)+f_{s,2}(h)\sin 2\left(\frac{2\pi}{P}t\right),$$

where h is the hour of the solar day, P is the mean period of a lunation in solar days, and t is the age of the moon in solar days; $f_{c,2}(h)$ and $f_{s,2}(h)$ are the observed variations at new moon and one-eighth phase respectively. This formula, it will be noticed, expresses the lunar variation as, in reality, a solar diurnal variation (h, the solar time, being the variable) which merely runs through a cycle of change depending on the age of the moon. This, in fact, was CHAMBERS' view—he termed the variation "luni-solar." It will be seen later, however, that there is a true lunar semi-diurnal variation which remains unchanged throughout the course of a lunation, as well as luni-solar components governed by the position of both bodies. As to CHAMBERS' expression for the variation, while it is numerically correct, it does not aid in interpreting the phenomenon, because it depends on two complex curves $f_{c,2}(h)$ and $f_{s,2}(h)$, for which no analytical expression was obtained; these two curves are not independent, as will appear later.

§ 8. FIGEE determined the harmonic coefficients of the diurnal and semi-diurnal components of the variation at each lunar phase, and came to the conclusion that "a regular variation of the movement of the magnetic needle with the moon's phases is not indicated by the observations at Batavia."‡ It will be shown, on the contrary, that the Batavian observations agree with those made at other places in manifesting considerable regularity of change with lunar phase.

§ 9. MOOS§ has made the valuable suggestion that the luni-solar variation may be regarded as a simple lunar variation the amplitude of "part of which goes through a series of wave-like changes in the course of a lunation." He multiplies each hourly value of the mean lunar variation determined from a whole month by $1+\cos(t+\nu)$, where t is the lunar time reckoned from upper culmination (one hour equalling 15°), and ν is the angular measure of the moon's age, reckoned as 0° at new moon, and changing through 360° in the course of a month. Curves showing the results of this calculation are exhibited for comparison with the observed curves, for the eight lunar phases, for the element of declination. The general similarity of the two sets of curves is sufficiently striking to show that the suggestion is in the right direction. It will be seen that this idea is, formally, much akin to SCHUSTER's idea of variable

* 'Batavian Observations,' XXVI., Appendix, p. 195, § 44.

† CHAMBERS, 'Phil. Trans.,' A, vol. 178 (1887).

‡ 'Batavian Observations,' XXVI., Appendix, § 44.

§ 'Bombay Magnetical Observations,' 1846–1905, vol. II., § 526.

conductivity, and is most naturally interpreted in that way. Moos, however, seems not to have thought of the matter in this simple light, but speaks of changes in the radio-activity of the earth's crust, due to a tidal action, as possibly responsible for the luni-solar changes, perhaps by ionizing the atmosphere indirectly; and also of the reflection by the moon of ionizing radiation from the sun.*

§ 10. Since the mean variation of any element over a whole lunation is almost exactly a semi-diurnal wave, Moos's expression is equivalent to

$$2\cos(2t+t_0)[1+\cos(t+\nu)] = \cos(t+t_0-\nu)+2\cos(2t+t_0)+\cos(3t+t_0+\nu), \quad . \quad \text{(A)}$$

though he did not himself write it out formally thus. The examination of the data by harmonic analysis, which is effected in the third part of this paper, is the best means of numerically testing Moos's suggestion, being preferable to a mere comparison of two sets of curves by eye. The desire to apply this test partly occasioned the present re-examination of the existing data, which also has in view the comparison of the results of these past determinations of the lunar magnetic variation (on which enormous labour has been spent) to see how far they confirm one another, and gauge the possibility of obtaining accurate information from them.

Moos's suggestion implies the presence, in the lunar diurnal variation at a particular lunar phase, of first and third harmonic components of amplitude equal to half that of the semi-diurnal component, and with phase angles which respectively decrease and increase by 45° with each change of lunar phase, the epoch of the second component remaining constant. No other relations or components would satisfy the above equation.

§ 11. The calculations from the observational data show that while first and third harmonic components possessing the above phase relations are present, the amplitudes are not generally in accordance with Moos's equation. Moreover, a fourth harmonic component, which was calculated in the first instance merely because to do so involved scarcely any trouble after the other components had been computed, was also found to be present, of quite appreciable amount, and obeying an unexpected phase law; its phase angle increases during each lunation by 4π, twice the amount of change in the phases of the first and third components.

There is considerable accidental error in the determinations of the phase angles and amplitudes at each lunar phase, as, of course, the material is much subdivided. While, however, the phase angles go through an easily recognizable monthly cycle, the amplitudes show no regular variation with lunar phase (the mean of a number of lunations is dealt with, of course, so that perigee and apogee occur at different phases during the period). The mean of the amplitudes at the separate phases gives, therefore, the best determination of the amplitudes of the first, third, and fourth

* 'Bombay Magnetical Observations,' 1846–1905, vol. II., § 527. It may be mentioned that earlier investigators had regarded the lunar variations as possibly due to the direct or indirect action of induced magnetism in the moon, arising from solar or terrestrial magnetism, or both.

components, as well as of the second; and similarly, by correcting the separate phase angles by the amount indicated by the regular phase law, and taking their mean, the accidental error of the determined phase angle at any particular lunar phase can be much reduced. In this way, as described more fully in § 27, the expression of the lunar variation at every period of the lunation, complete as far as the fourth harmonic term, is obtained. It is found that the amplitudes of the first and third harmonics are often unequal; sometimes their amplitude exceeds that of the second component, but generally they are less, down to about half this amount. The determined values of C and t_0 in the formula

$$C_1 \cos(t+t_0'-\nu)+C_2 \cos(2t+t_0'')+C_3 \cos(3t+t_0'''+\nu)+C_4 \cos(4t+t_0''''+2\nu), \qquad (B)$$

which has been found to fit the observations, are given in Tables XI., XII., and XIII. for all the stations and elements for which data were available. Moos's representation, it is seen, though it pointed in the right direction, is of too simple a character to represent the phenomenon; the solar excitation which it indicates is a matter which concerns the whole earth, and this action cannot be represented by a simple harmonic factor at each individual station.

§ 12. SCHUSTER* has calculated the effect of an atmospheric oscillation with a velocity potential Q_2^2 (which is also the main component of a lunar diurnal tide)† in producing, under the influence of a variable conductivity of amount

$$\rho = \rho_0(1+\gamma \cos \omega), \qquad \gamma \leqq 1$$

(where ω is the zenith distance of the sun from each particular point on the earth's surface), magnetic variations of one, two, three and more periods in the solar day. Adopting the rather more general expression

$$\rho = \rho_0[1+\gamma' \cos \theta+\gamma \sin \theta \cos(\lambda+t)], \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (C)$$

where θ is the colatitude, λ is the longitude, and $\lambda+t$ is the local time, he finds that the resulting magnetic potential (apart from a constant factor) is of the form

$$\sum_{\sigma=0}^{\infty} \frac{n+1}{2n+1} p_n^\sigma Q_n^\sigma \sin\{\sigma(\lambda+t)-\alpha\}+\sum_{\sigma=1}^{\infty} \frac{n+1}{2n+1} q_n^\sigma Q_n^\sigma \sin\{\sigma(\lambda+t)+\alpha\}, \quad . \quad (D)$$

where $Q_m^\tau \sin\{\tau(\lambda+t)-\alpha\}$ is the velocity potential. The coefficients p_n^σ and q_n^σ are numerical constants which depend on ν and ν'; their values are tabulated in the paper referred to.

It is shown in Part II. of the present paper that the above equation (D) holds good, whatever be the functional relation between ρ and ω, and this calculation is

* 'Phil. Trans.,' A, vol. 208, p. 163.

† Q_n^σ represents the tesseral function $\sin^\sigma \theta d^\sigma P_n/d\mu^\sigma$, where P_n is the zonal harmonic of degree n.

adapted, in § 23, to cover the case of the luni-solar magnetic variations. It is there shown that the equivalent expression to (D) is in this case (apart from a constant factor)

$$\sum_{\sigma=0}^{\infty} \frac{n+1}{2n+1} p_n^\sigma Q_n^\sigma \sin\{\sigma(\lambda+t')-\alpha+(\sigma-2)\nu\}$$
$$+\sum_{\sigma=0}^{\infty} \frac{n+1}{2n+1} q_n^\sigma Q_n^\sigma \sin\{\sigma(\lambda+t')+\alpha+(\sigma+2)\nu\}. \quad . \quad . \quad . \quad . \quad (E)$$

This expression, it should be noticed, consists of series of harmonic components of one, two, three, and more periods in the lunar day, with phase angles which depend on the age of the moon. In the second series the phase angles increase by $2(\sigma+2)\pi$ per lunation; this phase change is very rapid, even for the diurnal term, and with the lunation divided up into not more than eight parts, hardly comes within the range of observation, even if the coefficients q_n^σ were of the same order of magnitude as the p_n^σ coefficients. The theoretical values of q_n^σ are, however, much less than those of the important members of the p_n^σ set of coefficients, and therefore this part of the magnetic potential can be neglected. The other part consists of terms of period $2\pi\sigma$, whose phase angles increase by $2(\sigma-2)\pi$ per lunation; thus the phase of the first harmonic decreases by 2π each lunation, that of the second component remains constant, while the third, fourth, and higher components increase by amounts 2π, 4π, 6π, and so on. This, however, is exactly the law of phase change which is indicated by the formula (B), which was determined empirically from the observations.

At new moon, when $\nu = 0$, the formula indicates that all the harmonic components should have the same phase angle, or differ by 180 degrees exactly (since the coefficients may be of different sign). The data obtained in this paper show a very satisfactory agreement with this conclusion, when the extreme smallness of the whole phenomenon is considered.

§ 13. The amplitudes must next be considered. The actual calculations necessary for the comparison of theory and observation are given in § 25, and only the results obtained will be cited here. It appears that as regards the relative magnitudes of the first three components in the lunar variation, there is tolerably good agreement with the results derived either from SCHUSTER's simple theory $\rho/\rho_1 = 1+\cos\omega$, or from the more general theory of Part II. of this paper. The numerical constants ($\rho/\rho_0 = 1+3\cos\omega+\frac{9}{4}\cos^2\omega$) might be altered to fit the observations better, but it seems hardly worth while to do this till better observational material is available. The given constants were chosen to represent a function which should have a large maximum at midday, and should be small and nearly constant during the night hours.

§ 14. The deciding factor between the two expressions for ρ/ρ_0 is found to be the amplitude of the fourth harmonic component. Three tables are given in § 25 to illustrate this. They give the ratio of the amplitudes of the four harmonic components to that of the second component, for the three elements X, Y, Z. The first table is that calculated on the hypothesis $\rho/\rho_0 = 1+\cos\omega$, the second that calculated from $\rho/\rho_0 = 1+3\cos\omega+\frac{9}{4}\cos^2\omega$, and the third gives the observed values. The simple form of ρ gives altogether too small a value for C_4/C_2, while the second expression for ρ gives values of the right order, at any rate. Perhaps the detailed calculations in Part II. have not been carried to a sufficient degree of approximation, as the expressions for p_n^σ do not converge very rapidly. When better data are available, this point must receive consideration. Enough evidence, however, has been brought forward to show that the fourth harmonic component of the lunar variation favours the hypothesis that the conductivity during the night hours is small compared with its value during the daytime,* and that the rate of recombination of ions in the upper atmosphere (assuming this to be the seat of the effect) is rapid, as would naturally be expected.

The proper discussion of the observations, whether of the lunar or solar magnetic variations, can only be made on the basis of a reliable determination of the numerical coefficients of the various tesseral harmonics in the potential, derived from a number of observatories properly distributed over the globe. The significance of the lower harmonics in the lunar variation makes it desirable to obtain the terms in the potential down to those of the fourth type (Q_n^4)—not only for the lunar variation, but also for the solar variation; its fourth harmonic shows a sufficient degree of constancy, at most observatories, to entitle it to respect as having definite physical significance.

PART II.—*Mathematical Theory.*

§ 15. The problem in hand is to determine the current function of the electric currents induced in a spherical shell of fluid by its quasi-tidal motion across a radial magnetic field of force, the electric conductivity of the fluid at any point being a known function of the angular distance between that point and another (that with the sun at its zenith) which uniformly rotates round the axis of the sphere. The velocity potential ψ of the motion will be expressed as the sum of a number of terms such as

$$Q_m^\tau \sin(\tau\,.\,\overline{\lambda+t}-\alpha),$$

where Q_m^τ is a surface harmonic of degree m and type τ, and λ is the longitude measured towards the east from some standard meridian, at which the local time is t. The colatitude and zenith distance of the sun will be denoted by θ and ω respectively;

* It is not asserted that any observational evidence has been brought forward in favour of the particular numerical constants here chosen for ρ, but only that the observations indicate the presence of an appreciable term in ρ depending on $\cos 2\omega$, and that this term, if present, may be expected, on general physical grounds, to be of such a sign as to diminish the value of ρ at night as compared with the value by day.—June 11, 1913.

if δ is the declination of the sun, evidently we have

$$\cos\omega = \sin\delta\cos\theta + \cos\delta\sin\theta\cos(\lambda+t)$$
$$\equiv x+2y\mu,$$

where

$$x = \sin\delta\cos\theta, \qquad 2y = \cos\delta\sin\theta, \qquad \mu = \cos(\lambda+t).$$

The conductivity and resistivity at the point (θ, λ) will be denoted by ρ and κ' respectively, $\rho\kappa'$ being, of course, equal to unity. For the present we shall suppose that ρ and κ' are finite and continuous functions of ω, so that they can be expressed as FOURIER'S series in $\cos n\omega$ over the range $0, \pi$; ρ will certainly satisfy this condition, and the case of $\rho = 0$, $\kappa' = \infty$ will be considered later. Further, it will be assumed possible to express κ' as a TAYLOR'S series in $\cos\omega$, and it is in this form that we shall suppose the resistivity to be given, as one of the data of the problem. Theoretically this is a limitation of the problem, as there are some functions which cannot be expressed in the form stated; for instance, if the conductivity were proportional to $\cos\omega$ in that hemisphere on which the sun is shining, and zero or constant over the other hemisphere, κ' could not be so expressed. But in reality nothing of value is lost, as any continuous function can be approximately expressed in the form of a TAYLOR'S series to any desired degree of accuracy.

[Some further explanation of this use of series may be desirable. The series used in the analysis are all written as infinite ones, for the sake of formal simplicity and theoretical completeness. In the detailed execution of the work, however, only a finite number of these terms can be utilized, as workable general expressions for the coefficients in the current function R cannot be obtained. The actual procedure, therefore, must be to take a finite number of terms of the FOURIER'S series for ρ, transform this into a polynomial in $\cos\omega$ (this also, of course, will have only a finite number of terms), and work out the coefficients of R in terms of the coefficients of this polynomial to as great a degree of accuracy as is practicable and desirable. This is the course of the work in §§ 18–20, where the terms $(a+b\cos\omega+c\cos 2\omega)$ of the FOURIER'S series for ρ are taken, and the expression for R is worked out as far as concerns the terms in a, b, b^2, and c. The resistivity $1/\rho$ is introduced into the calculations for purely mathematical reasons, on account of certain analytical advantages which it seems to offer. The results obtained in this way, in terms of the coefficients of ρ, might be got otherwise by an extension of the method used by SCHUSTER. This identity of results is clear from the fact that if the FOURIER coefficients of ρ are small enough the TAYLOR'S series for $1/\rho$ is absolutely convergent, and the legitimacy of the use of $1/\rho$ is in this case immediately evident; the formal results, however, do not depend on any property of convergence, so that the results obtained by using $1/\rho$ remain equally valid with those obtained in any other way, even though the series for $1/\rho$ should become non-convergent. This is one of many instances in which it is possible and advantageous to use expressions which may become non-convergent to obtain results which can be got less simply in other ways. Whether the final result is convergent depends in this case only on ρ, and not on the processes of analysis used to deduce R from ρ.—*Added June* 11, 1913.]

We shall write, therefore,

$$\kappa = \mathrm{C}ae \sum_0^\infty d_p \cos^p \omega, \qquad \equiv \mathrm{C}ae\kappa,$$

where C, e, and a are constants (introduced for convenience) whose meaning will be explained later, and the coefficients d_p are given numbers. Expanding $\cos^p \omega$ in terms of μ, we have

$$\begin{aligned}\kappa &= \sum_0^\infty d_p (x+2y\mu)^p \\ &= \sum_0^\infty e_p \,.\, (2y)^p \,.\, \mu^p \\ &= f_0 + 2\sum_1^\infty f_p \cos p(\lambda+t),\end{aligned}$$

where

$$e_p = \sum_{l=0}^\infty {}_{l+p}\mathrm{C}_p \,.\, d_{l+p} \,.\, x^l,$$

and (since

$$2^{m-1}\mu^m = 2^{m-1}\cos^m(\lambda+t) = \cos m(\lambda+t) + {}_m\mathrm{C}_1 \cos(m-2)(\lambda+t) + {}_m\mathrm{C}_2 . \cos(m-4)(\lambda+t) + \ldots)$$

f_p is given (for $p \geqq 0$) by

$$\begin{aligned}f_p &= \sum_{q=0}^\infty {}_{p+2q}\mathrm{C}_q \,.\, e_{p+2q} \,.\, y^{p+2q}, \\ &\equiv y^p \sum_{q=0}^\infty \alpha_{p,q} y^{2q}.\end{aligned}$$

Here we have written

$$\alpha_{p,q} \equiv {}_{p+2q}\mathrm{C}_q e_{p+2q}.$$

In virtue of the definitions of e_p and $\alpha_{p,q}$, we have

$$\frac{de_p}{dx} = (p+1)\,e_{p+1}, \qquad \frac{d\alpha_{p,q}}{dx} = (q+1)\,\alpha_{p-1,q+1}.$$

Next we consider the differential coefficients of κ. We have

$$\begin{aligned}\frac{d\kappa}{d\lambda} &= -2\sum_1^\infty p f_p \sin p(\lambda+t), \\ \frac{d\kappa}{d\theta} &= \frac{df_0}{d\theta} + 2\sum_1^\infty \frac{df_p}{d\theta}\cos p(\lambda+t).\end{aligned}$$

We shall write f'_p for $\dfrac{df_p}{d\theta}$; evidently we have

$$f'_p = \tfrac{1}{2}y^{p-1}\cos\delta\cos\theta \sum_0^\infty (p+2q)\,\alpha_{p,q}y^{2q} - y^p \sin\delta\sin\theta \sum_0^\infty (q+1)\,\alpha_{p-1,q+1}y^{2q}.$$

§ 16. Following SCHUSTER's notation and treatment, the earth will be regarded as a uniformly magnetized sphere of radius a, whose magnetic potential may be resolved into the zonal harmonic of the first degree and the tesseral harmonic of the first degree and type. The former harmonic is much the larger of the two, as the inclination (ϕ) of the magnetic to the geographical axis is small. The radial force can be expressed as

$$V = C\cos\theta + C\tan\phi\sin\theta\cos\lambda,$$

where C is a constant not differing much from $-\frac{2}{3}$ (the force being measured positive outwards), and λ is now the longitude measured from the meridian (68° 31′ west of Greenwich) containing the magnetic axis.

The components of electric force, X and Y, measured towards the south and east respectively, are

$$Xa = \frac{V}{\sin\theta}\frac{d\psi}{d\lambda}, \qquad Ya = -V\frac{d\psi}{d\theta},$$

ψ being the velocity potential.

If we express X and Y in the form

$$X = \frac{dS}{d\theta} + \frac{\kappa'}{e}\frac{dR}{\sin\theta\, d\lambda}, \qquad Y = \frac{dS}{\sin\theta\, d\lambda} - \frac{\kappa'}{e}\frac{dR}{d\theta},$$

where κ' is the known resistivity and e the thickness of the conducting atmospheric shell, the function R will be the current function of the electric currents produced by X and Y (neglecting electric inertia). The function S is the potential of a system of electric forces which in the steady state are balanced by a static distribution of electricity revolving round the earth, and causing a variation in the electrostatic potential which is found to be too weak to affect our instruments.

To determine R we shall eliminate S, thus obtaining the equation

$$(1) \qquad \frac{dX}{d\lambda} - \frac{d}{d\theta}\left(Y\sin\theta\right) = \frac{1}{\sin\theta}\frac{d}{d\lambda}\left(Ca\kappa\frac{dR}{d\lambda}\right) + \frac{d}{d\theta}\left(Ca\kappa\sin\theta\frac{dR}{d\theta}\right).$$

Instead of using the resistivity κ', SCHUSTER worked with the conductivity ρ (using the special form $1+k\cos\omega$), in order to avoid the difficulties introduced by "the high and possibly infinite values which κ' would take when the conductivity sinks low or vanishes."* These difficulties, however, are found not to be serious, and the work is greatly simplified by the use of κ', which enables R to be determined directly, without first evaluating S, as is necessary when ρ is kept as the variable quantity. The investigation can also be made much more general, without formal complexity, when κ' is used.

* SCHUSTER, 'Phil. Trans.,' A, vol. 208, p. 190.

Recalling the expressions for X, Y in terms of the velocity potential ψ, the left-hand side of the last equation, after division by Ca sin θ, may be written*

$$(2)\quad -\frac{1}{2m+1}\left[m(m+2)(m-\tau+1)\,Q_{m+1}{}^{\tau}+(m-1)(m+1)(m+\tau)\,Q_{m-1}{}^{\tau}\right]\sin(\tau\,.\,\overline{\lambda+t}-\alpha)$$

$$+\frac{\tan\phi}{2(2m+1)}\left[\{(m-1)(m+1)\,Q_{m-1}{}^{\tau+1}-m(m+2)\,Q_{m+1}{}^{\tau+1}\}\sin(\tau\,.\,\overline{\lambda+t}+\lambda-\alpha)\right.$$

$$+\{-(m-1)(m+1)(m+\tau)(m+\tau-1)\,Q_{m-1}{}^{\tau-1}$$

$$\left.+m(m+2)(m-\tau+1)(m-\tau+2)\,Q_{m+1}{}^{\tau-1}\}\sin(\tau\,.\,\overline{\lambda+t}-\lambda-\alpha)\right].$$

The right-hand side of our equation for R, after division by Ca sin θ, becomes equal to

$$(3)\qquad \frac{\kappa}{\sin^2\theta}\left(\frac{d^2}{d\lambda^2}+\sin\theta\frac{d}{d\theta}\sin\theta\frac{d}{d\theta}\right)\mathrm{R}+\left(\frac{d\mathrm{R}}{\sin\theta\,d\lambda}\cdot\frac{d\kappa}{\sin\theta\,d\lambda}+\frac{d\mathrm{R}}{d\theta}\cdot\frac{d\kappa}{d\theta}\right).$$

We suppose R to be expressed as the sum of a number of tesseral harmonics $p_n{}^{\sigma}Q_n{}^{\sigma}\sin(\sigma\lambda'-\alpha')$, where $p_n{}^{\sigma}$ is a numerical coefficient, λ' has been written for $\lambda+t$, and σ ranges (possibly) from $-\infty$ to $+\infty$. The contribution of each such term to the total value of the last expression is easily seen to be the product of $p_n{}^{\sigma}$ into

$$-n(n+1)\,Q_n{}^{\sigma}\sin(\sigma\lambda'-\alpha')\,\{f_0+2\sum_1^{\infty}f_p\cos p\lambda'\},$$

$$-\frac{2\sigma Q_n{}^{\sigma}}{\sin^2\theta}\cos(\sigma\lambda'-\alpha')\sum_1^{\infty}pf_p\sin p\lambda',$$

$$+2\frac{dQ_n{}^{\sigma}}{d\theta}\sin(\sigma\lambda'-\alpha')\,\{f'_0+2\sum_1^{\infty}f'_p\cos p\lambda'\},$$

where we have inserted the values of κ and its differential coefficients, and have transformed the first line by means of LAPLACE's equation

$$\sin\theta\frac{d}{d\theta}\sin\theta\frac{dQ_n{}^{\sigma}}{d\theta}+\frac{d^2Q_n{}^{\sigma}}{d\lambda^2}+n(n+1)\sin^2\theta\,.\,Q_n{}^{\sigma}=0.$$

So far f_p and f'_p have been defined only for positive and zero values of p; we now extend the definition by the equations

$$f_p=f_{-p},\qquad f'_p=f'_{-p}.$$

* *Ibid.*, pp. 188, 189.

This enables us to write expression (3) in the form

$$-n(n+1)\,Q_n^\sigma \sum_{-\infty}^{+\infty} f_p \sin(\overline{\sigma+p}\,.\,\lambda'-\alpha'),$$

$$-\frac{\sigma Q_n^\sigma}{\sin^2\theta}\sum_{-\infty}^{+\infty} pf_p \sin(\overline{\sigma+p}\,.\,\lambda'-\alpha'),$$

$$+\frac{dQ_n^\sigma}{d\theta}\sum_{-\infty}^{+\infty} f'_p \sin(\overline{\sigma+p}\,.\,\lambda'-\alpha'),$$

$$= \sum_{p=-\infty}^{+\infty} R_n^\sigma(p) \sin(\overline{\sigma+p}\,.\,\lambda'-\alpha'),$$

where

$$R_n^\sigma(p) \equiv \left(-n(n+1)\,Q_n^\sigma - \frac{p\sigma Q_n^\sigma}{\sin^2\theta}\right) f_p + f'_p \frac{dQ_n^\sigma}{d\theta}.$$

When p is positive, substituting our expressions for f_p and f'_p, we find

$$R_n^\sigma(p) = \sum_{q=0}^{\infty} \alpha_{p,q} y^{2q}\,.\left[-n(n+1)\,Q_n^\sigma - \frac{p\sigma Q_n^\sigma}{\sin^2\theta}\right] y^p,$$

$$+\sum_{q=0}^{\infty}(p+2q)\,\alpha_{p,q} y^{2q}\,.\,\tfrac{1}{2}\cos\delta\cos\theta\frac{dQ_n^\sigma}{d\theta}\,.\,y^{p-1},$$

$$-\sum_{q=0}^{\infty}(q+1)\,\alpha_{p-1,q+1} y^{2q}\,.\,\sin\delta\sin\theta\frac{dQ_n^\sigma}{d\theta}\,.\,y^p,$$

$$= \tfrac{1}{2}\cos\delta\sum_{q=0}^{\infty}\alpha_{p,}\;y^{2q}\left[(p+q)\left(\cos\theta\frac{dQ_n^\sigma}{d\theta}-\frac{\sigma Q_n^\sigma}{\sin\theta}\right)+q\left(\cos\theta\frac{dQ_n^\sigma}{d\theta}+\frac{\sigma Q_n^\sigma}{\sin\theta}\right)\right.$$
$$\left.-n(n+1)\,Q_n^\sigma\sin\theta\right]y^{p-1},$$

$$-\sin\delta\sum_{q=0}^{\infty}(q+1)\,\alpha_{p-1,q+1}y^{2q}\,.\,\sin\theta\frac{dQ_n^\sigma}{d\theta}\,.\,y^p.$$

Since, in the original equation for $R_n^\sigma(p)$, a change in the sign of p only affects the term $\sigma Q_n^\sigma/\sin\theta$ in the first term, from our last expression we may at once write down the value of $R_n^\sigma(-p)$, p being positive, by changing the sign of $\sigma Q_n^\sigma/\sin\theta$. Thus,

$$R_n^\sigma(-p) = \tfrac{1}{2}\cos\delta\sum_{q=0}^{\infty}\alpha_{p,q}y^{2q}\left[(p+q)\left(\cos\theta\frac{dQ_n^\sigma}{d\theta}+\frac{\sigma Q_n^\sigma}{\sin\theta}\right)+q\left(\cos\theta\frac{dQ_n^\sigma}{d\theta}-\frac{\sigma Q_n^\sigma}{\sin\theta}\right)\right.$$
$$\left.-n(n+1)\,Q_n^\sigma\sin\theta\right]y^{p-1},$$

$$-\sin\delta\sum_{q=0}^{\infty}(q+1)\,\alpha_{p-1,q+1}y^{2q}\sin\theta\frac{dQ_n^\sigma}{d\theta}\;y^p.$$

§ 17. For convenience and clearness, some well-known formulæ of transformation will now be set down. These have been much used by SCHUSTER in his papers, and the equations, and some formulæ derived from them, will be denoted by the same Roman letters which he uses.*

$$(2n+1)\cos\theta\, Q_n^{\sigma} = (n-\sigma+1)\, Q_{n+1}^{\sigma}+(n+\sigma)\, Q_{n-1}^{\sigma}, \quad \text{(A)}$$

$$(2n+1)\sin\theta\, Q_n^{\sigma} = Q_{n+1}^{\sigma+1}-Q_{n-1}^{\sigma+1}, \quad \text{(B)}$$

$$= (n+\sigma)(n+\sigma-1)Q_{n-1}^{\sigma-1}-(n-\sigma+2)(n-\sigma+1)Q_{n+1}^{\sigma-1}, \quad \text{(C)}$$

$$\frac{2\sigma Q_n^{\sigma}}{\sin\theta} = (n+\sigma)(n+\sigma-1)\, Q_{n-1}^{\sigma-1}+Q_{n-1}^{\sigma+1}, \quad \text{(D)}$$

$$= Q_{n+1}^{\sigma+1}+(n-\sigma+2)(n-\sigma+1)\, Q_{n+1}^{\sigma-1}, \quad \text{(E)}$$

$$\frac{2dQ_n^{\sigma}}{d\theta} = (n+\sigma)(n-\sigma+1)\, Q_n^{\sigma-1}-Q_n^{\sigma+1}, \quad \text{(F)}$$

$$(2n+1)\left(\cos\theta\frac{dQ_n^{\sigma}}{d\theta}-\frac{\sigma Q_n^{\sigma}}{\sin\theta}\right) = -\{nQ_{n+1}^{\sigma+1}-(n+1)\, Q_{n-1}^{\sigma+1}\},$$

$$(2n+1)\left(\cos\theta\frac{dQ_n^{\sigma}}{d\theta}+\frac{\sigma Q_n^{\sigma}}{\sin\theta}\right) = \{n(n-\sigma+2)(n-\sigma+1)\, Q_{n+1}^{\sigma-1} + (n+1)(n+\sigma)(n+\sigma-1)\, Q_{n-1}^{\sigma-1}\},$$

$$(2n+1)\sin\theta\frac{dQ_n^{\sigma}}{d\theta} = n(n-\sigma+1)\, Q_{n+1}^{\sigma}-(n+1)(n+\sigma)\, Q_{n-1}^{\sigma}.$$

Making use of these equations, we obtain the following expressions :—

$$R_n^{\sigma}(p) = \frac{\cos\delta}{2(2n+1)}\sum_{q=0}^{\infty}\alpha_{p,q}y^{2q}\left[-\{n(n+p+q+1)\, Q_{n+1}^{\sigma+1}+(n+1)(n-p-q)\, Q_{n-1}^{\sigma+1}\} + q\{n(n-\sigma+2)(n-\sigma+1)\, Q_{n+1}^{\sigma-1}+(n+1)(n+\sigma)(n+\sigma-1)\, Q_{n-1}^{\sigma-1}\}\right]y^{p-1} - \frac{\sin\delta}{2n+1}\sum_{q=0}^{\infty}(q+1)\,\alpha_{p-1,q+1}y^{2q}\left[n(n-\sigma+1)\, Q_{n+1}^{\sigma}-(n+1)(n+\sigma)\, Q^{\sigma}\right]y^{p}.$$

In $R_n^{\sigma}(-p)$, the second term remains the same, while the expression in square brackets in the first term becomes

$$[\{n(n+p+q+1)(n-\sigma+2)(n-\sigma+1)\, Q_{n+1}^{\sigma-1}-(n-p-q)(n+1)(n+\sigma)(n+\sigma-1)\, Q_{n-1}^{\sigma-1}\} - q\{nQ_{n+1}^{\sigma+1}+(n+1)\, Q_{n-1}^{\sigma+1}\}].$$

These expressions for $R_n^{\sigma}(\pm p)$ are of the type

$$\alpha y^{2q}Q_{\nu}^{\sigma+1}\cdot y^{p-1}, \qquad \alpha y^{2q}Q_{\nu}^{\sigma}y^{p}, \qquad \alpha y^{2q}Q_{\nu}^{\sigma-1}y^{p-1}.$$

Now by equations (B) and (C), it is evident that $Q_{\nu}^{\sigma+1}y^{p-1}$, $Q_{\nu}^{\sigma-1}y^{p-1}$, and $Q_{\nu}^{\sigma}y^{p}$ can be expressed as the sum of a number of tesseral harmonics all of type $\sigma+p$ or all of type $\sigma-p$ (at will), and of degrees ranging, by steps of 2, from $\nu\pm(p-1)$, $\nu\pm(p-1)$ and $\nu\pm p$ respectively. Further multiplication by y^{2q} can be so arranged as

* *Ibid.*, pp. 187–189.

to leave the type unchanged, while extending the range of the degrees by $4q$. Also by equation (A), the coefficient α, which, it will be remembered, is a power series in $\cos\theta$, leaves the type unchanged while it increases the range of the degrees of the resulting tesseral harmonics. In every case, therefore (p positive or negative), R_n^p can be expressed as the sum of a number of terms such as $Q_{n'}^{\sigma+p}$. Therefore if we write Ψ for the sum of all the expressions (2) resulting from each term $Q_m^\tau \sin(\tau\lambda'-\alpha)$ in the velocity potential ψ, the fundamental equation (1) for R takes the form

$$(4) \qquad \Psi = \sum_{p=-\infty}^{+\infty}\{\sum_{n',\sigma} k_{n'}^{\sigma,p} Q_{n'}^{\sigma+p} \sin(\overline{\sigma+p}\,.\,\lambda'-\alpha')\},$$

where $k_{n'}^{\sigma,p}$ is a coefficient whose value can be determined in terms of p_n^σ and the coefficients d_p in the TAYLOR'S series for κ'. By equating the coefficients of harmonics of the same degree and type, on the two sides of the equation, we obtain equations to determine the coefficient p_n^σ in terms of the d_p's and the known constants of the velocity potential. In practice this must be done by a process of successive approximation. Knowing, from the form of the above equation, which is linear in p_n^σ and d_p, that every coefficient p_n^σ can be expressed as a TAYLOR'S series in $\frac{d_0}{d_0}, \frac{d_1}{d_0}, \frac{d_2}{d_0}$, and so on, we can determine this series by successively assuming that all save one particular variable $\frac{d_p}{d_0}$ are zero, and considering this variable alone, it may easily be seen that the phase angle of every term in R arising from a particular term in Ψ is the same as that of the latter.

§ 18. SCHUSTER has worked out the values of p_n^σ for the special form of conductivity already mentioned, and for the two terms $Q_1^1 \sin(\lambda'-\alpha)$ and $Q_2^2 \sin(2\lambda'-\alpha)$ in the velocity potential, to the fourth order of approximation, and he finds that the numerical coefficients of the terms are such that only the first order term (depending, in our notation, on d_1/d_0) are large enough to be detectable by observation. The present calculation will not be carried so far, therefore, and will not include terms of higher order than d_2/d_0 or $(d_1/d_0)^2$. Also, since in the expression for Ψ the term depending on the inclination of the magnetic to the geographical axis is multiplied by the small factor $\tan\phi$, the part of R depending on this term will only be calculated as far as the first order d_1/d_0. Further, since the actual atmospheric oscillations seem to be mainly performed in the simplest mode possible, so that $m = \tau$ for the principal terms, the second order terms will be neglected for the smaller harmonics in the velocity potential ψ, for which $m \neq \tau$.

We therefore consider the terms in R which depend upon a term $A_{m'}^{\tau'} Q_{m'}^{\tau'}$ in Ψ, where m' and τ' are quite general, except that in the terms depending on d_2/d_0 we shall suppose $m' = \tau'+1$ (since $Q_{m'}^{\tau'} = 0$ when $\tau' > m'$, the term in (2) depending on Q_{m-1}^{τ} vanishes when $m = \tau$).

It will first be necessary to write out the developed expressions for $R_n^\sigma(0)$, $R_n^\sigma(\pm 1)$, $R_n^\sigma(\pm 2)$ as far as the terms in d_2. No other values of p in $R_n^\sigma(p)$ give terms containing d_0, d_1, d_2. It is convenient to write down first of all the values of $f_0, f_1, f_2, f'_0, f'_1, f'_2$, omitting d_3 and higher terms.

$$\begin{aligned} f_0 &= e_0 + 2e_2y^2, & e_0 &= d_0 + d_1x + d_2x^2, \\ f_1 &= e_1y, & e_1 &= d_1 + 2d_1x, \\ f_2 &= e_2y^2, & e_2 &= d_2, \\ f'_0 &= 2e_2y \cos\delta \cos\theta - e_1 \sin\delta \sin\theta, \\ f'_1 &= \tfrac{1}{2}e_1 \cos\delta \cos\theta - 2e_2y \sin\delta \sin\theta, \\ f'_2 &= e_2y \cos\delta \cos\theta. \end{aligned}$$

$$\begin{aligned} R_n^\sigma(0) = &-n(n+1)\, d_0 Q_n^\sigma \\ &- \frac{d_1 \sin\delta}{2n+1}\left[2n(n+2)\,Q_{n+1}^\sigma + (n-1)(n+1)(2n-1)\,Q_{n-1}^\sigma\right] \\ &- \frac{d_2 \sin^2\delta}{(2n+1)(2n+3)}\left[6n(n+3)\,Q_{n+2}^\sigma + (6n^3+13n^2-7n-6)\,Q_n^\sigma\right] \\ &+ \frac{d_2 \cos^2\delta}{(2n+1)(2n+3)}\left[3n(n+3)\,Q_{n+2}^\sigma + \{4n^2 - n^2(n+1)(2n+1) \right. \\ &\qquad\left. -(n+1)(2n+3)\}\,Q_n^\sigma\right]. \end{aligned}$$

As already stated, in the last two lines we have substituted $\sigma = n-1$ in working out the numerical coefficients, as also in all other terms with d_2 as factor.

$$\begin{aligned} R_n^\sigma(1) = &\frac{d_1 \cos\delta}{2(2n+1)}\left[-n(n+2)\,Q_{n+1}^{\sigma+1} + (n^2-1)\,Q_{n-1}^{\sigma+1}\right] \\ &+ \frac{d_2 \cos\delta \sin\delta}{(2n+1)(2n+3)}\left[-2n(n+3)\,Q_{n+2}^{\sigma+1} - (2n^3+3n^2-5n-3)\,Q_n^{\sigma+1}\right] \end{aligned}$$

$$\begin{aligned} R_n^\sigma(-1) = &\frac{d_1 \cos\delta}{2(2n+1)}\left[n(n+2)(n-\sigma+2)(n-\sigma+1)\,Q_{n+1}^{\sigma-1} \right. \\ &\qquad\left. -(n-1)(n+1)(n+\sigma)(n+\sigma-1)\,Q_{n-1}^{\sigma-1}\right] \\ &+ \frac{d_2 \cos\delta \sin\delta}{(2n+1)(2n+3)}\left[24n(n+3)\,Q_{n+2}^{\sigma-1} + \{2n(2n-1)(n+6) \right. \\ &\qquad\left. -2(n+1)(2n+3)(2n^2-4n+3)\}\,Q_n^{\sigma-1}\right] \end{aligned}$$

$$R_n^\sigma(2) = -\frac{n(n+3)}{4(2n+1)(2n+3)} d_2 \cos^2\delta \, . \, Q_{n+2}^{\sigma+2}$$

$$\begin{aligned} R_n^\sigma(-2) = &\frac{d_2 \cos^2\delta}{(2n+1)(2n+3)}\left[-30n(n+3)\,Q_{n+2}^{\sigma-2} \right. \\ &+ \{3n(n+3)(2n-1)(n-1) \\ &\qquad + 3(n-1)(n-2)(n+1)(2n+1)\}\,Q_n^{\sigma-2} \\ &\left. -(n-2)^2(n^2-1)(2n-3)(2n+1)\,Q_{n-2}^{\sigma-2}\right]. \end{aligned}$$

If in the above expressions the term Q_n^{-1} occurs, it may be replaced by $-Q_n^1/n(n+1)$.

§ 19. Consider now the terms arising from a term $A_{m'}^{\tau'}Q_{m'}^{\tau'}\sin(\tau'\lambda'-\alpha)$ in Ψ. The only term of corresponding type involving d_0, on the right-hand side of equation (4), which does not vanish, is

$$-p_{m'}^{\tau'}m'(m'+1)\,d_0 Q_{m'}^{\tau'}\sin(\tau'\lambda'-\alpha').$$

Consequently, to order $1/d_0$,

$$p_{m'}^{\tau'} = -\frac{A_{m'}^{\tau'}}{m'(m'+1)\,d_0}, \quad \text{and} \quad \alpha' = \alpha,$$

and no other term is of order $1/d_0$. Next, taking terms of order d_1/d_0, it is evident that these can only arise from $R_{m'}^{\tau'}(0)$, $R_{m'}^{\tau'}(\pm 1)$, which involve $d_1 Q_{m'\pm 1}^{\tau'}$, $d_1 Q_{m'\pm 1}^{\tau'\pm 1}$. Equating the sum of the coefficients of the terms containing these harmonics, with factors d_0 and d_1, to zero, we get the following general expressions for $p_{m'\pm 1}^{\tau'}$, $p_{m'\pm 1}^{\tau'\pm 1}$, to order d_1/d_0:—

$$(5)\quad \left\{\begin{aligned}
p_{m'+1}^{\tau'} &= -\frac{d_1\sin\delta}{d_0}\,\frac{m'(m'-\tau'+1)}{(m'+1)(2m'+1)}\,p_m^{\tau'},\\
p_{m'-1}^{\tau'} &= -\frac{d_1\sin\delta}{d_0}\,\frac{(m'+1)(m'+\tau')}{m'(2m'+1)}\,p_{m'}^{\tau'},\\
p_{m'+1}^{\tau'+1} &= -\tfrac{1}{2}\frac{d_1\cos\delta}{d_0}\,\frac{m'}{(m'+1)(2m'+1)}\,p_{m'}^{\tau'},\\
p_{m'-1}^{\tau'+1} &= +\tfrac{1}{2}\frac{d_1\cos\delta}{d_0}\,\frac{m'+1}{m'(2m'+1)}\,p_{m'}^{\tau'},\\
p_{m'+1}^{\tau'-1} &= +\tfrac{1}{2}\frac{d_1\cos\delta}{d_0}\,\frac{m'(m'-\tau'+2)(m'-\tau'+1)}{(m'+1)(2m'+1)}\,p_{m'}^{\tau'},\\
p_{m'-1}^{\tau'-1} &= -\tfrac{1}{2}\frac{d_1\cos\delta}{d_0}\,\frac{(m'+1)(m'+\tau')(m'+\tau'-1)}{m'(2m'+1)}\,p_{m'}^{\tau'}.
\end{aligned}\right.$$

If the type of any of these coefficients exceeds the degree, it must be set equal to zero. No other terms are of order d_1/d_0.

So far no restriction has been placed on m' and τ'. In making a further approximation, we shall not write out general expressions, but shall consider the effect of the second order terms (d_2/d_0 and d_1^2/d_0^2) on two specific terms in the velocity potential of the atmosphere, viz., $Q_1^1\sin(\lambda'-\alpha_1)$ and $Q_2^2\sin(2\lambda'-\alpha_2)$, which give rise in Ψ to terms

$$-Q_2^1\sin(\lambda'-\alpha_1) \quad \text{and} \quad -\tfrac{12}{5}Q_3^2\sin(2\lambda'-\alpha_2).$$

The terms of the proper order on the right-hand side of (4) are (*a*) those involving d_2 from $R_{m'}^{\tau'}(\pm 2)$, and (*b*) those involving d_1^2/d_0^2 from $R_{m'\pm 1}^{\tau'}(p)$ and $R_{m'\pm 1}^{\tau'\pm 1}(p)$ where $p = 0, \pm 1$.

Considering the diurnal variation first, the terms (*a*) are found, from the formulæ at the foot of p. 296, to be

$$\tfrac{1}{7}d_2p_2^1\,[(6Q_4^1-13Q_2^1)\cos^2\delta\sin(\lambda'-\alpha_1)-(12Q_4^1+16Q_2^1)\sin^2\delta\sin(\lambda'-\alpha_1)$$
$$-\{(4Q_4^2+3Q_2^2)\sin(2\lambda'-\alpha_1)-(48Q_4^0-6Q_2^0)\sin(-\alpha_1)\}\sin\delta\cos\delta$$
$$-\tfrac{1}{2}Q_4^3\cos^2\delta\sin(3\lambda'-\alpha_1)+3\,(Q_4^1-Q_2^1)\cos^2\delta\sin(-\lambda'-\alpha_1)],$$

and the terms (*b*) are

$$p_3^1\,d_1\,[\{-(\tfrac{15}{14}Q_4^2-\tfrac{4}{7}Q_2^2)\sin(2\lambda'-\alpha_1)+(\tfrac{90}{7}Q_4^0-\tfrac{48}{7}Q_2^0)\sin(-\alpha_1)\}\cos\delta$$
$$-(\tfrac{45}{7}Q_4^1+\tfrac{32}{7}Q_2^1)\sin\delta\sin(\lambda'-\alpha_1)],$$
$$+p_1^1\,d_1\,[\{-\tfrac{1}{2}Q_2^2\sin(2\lambda'-\alpha_1)+Q_2^0\sin(-\alpha_1)\}\cos\delta-Q_2^1\sin\delta\sin(\lambda'-\alpha_1)],$$
$$+p_3^2\,d_1\,[\{-\tfrac{15}{14}Q_4^3\sin(3\lambda'-\alpha_1)+(\tfrac{45}{7}Q_4^1-\tfrac{80}{7}Q_2^1)\sin(\lambda'-\alpha_1)\}\cos\delta$$
$$-(\tfrac{30}{7}Q_4^2+\tfrac{40}{7}Q_2^2)\sin\delta\sin(2\lambda'-\alpha_1)],$$
$$+p_3^0\,d_1\,[(-\tfrac{15}{14}Q_4^1+\tfrac{4}{7}Q_2^1)\{\sin(\lambda'-\alpha_1)+\sin(-\lambda'-\alpha_1)\}\cos\delta$$
$$-(\tfrac{60}{7}Q_4^0+\tfrac{24}{7}Q_2^0)\sin\delta\sin(-\alpha_1)],$$
$$+p_1^0\,d_1\,[\{-\tfrac{1}{2}Q_2^1\sin(\lambda'-\alpha_1)-\tfrac{1}{2}Q_2^1\sin(-\lambda'-\alpha_1)\}\cos\delta-2Q_2^0\sin\delta\sin(-\alpha_1)].$$

The sum of the coefficients of any particular term $Q_n^\sigma \sin(\sigma\lambda'-\alpha_1)$ in the above, must be equated with $p_n^\sigma n(n+1)\,Q_n^\sigma\sin(\sigma\lambda'-\alpha_1)$. The values of p_n^σ thus calculated are given in Table I. It should be remarked that Q_n^{-1} has been replaced by $-Q_n^1/n\,(n+1)$, and the coefficient of $Q_n^1\sin(\lambda'+\alpha_1)$ will be denoted by q_n^1.

§ 20. The second order terms arising from the semi-diurnal atmospheric oscillation are similarly written down as follows:—

(*a*) $$p_3^2\,d_2\,[\{(\tfrac{6}{7}Q_5^2-4Q_3^2)\cos^2\delta-(\tfrac{12}{7}Q_5^2+4Q_3^2)\sin^2\delta\}\sin(2\lambda'-\alpha_2)$$
$$+\{(-\tfrac{4}{7}Q_5^3-Q_3^3)\sin(3\lambda'-\alpha_2)+(\tfrac{48}{7}Q_5^1-6Q_3^1)\sin(\lambda'-\alpha_2)\}\sin\delta\cos\delta$$
$$-\tfrac{1}{14}Q_5^4\cos^2\delta\sin(4\lambda'-\alpha_2)+(12Q_3^0-\tfrac{60}{7}Q_5^0-\tfrac{24}{7}Q_1^0)\cos^2\delta\sin(-\alpha_2)],$$

(*b*) $$p_2^1d_1\,[\{-\tfrac{4}{5}Q_3^2\sin(2\lambda'-\alpha_2)+(\tfrac{24}{5}Q_3^0-\tfrac{9}{5}Q_1^0)\sin(-\alpha_2)\}\cos\delta$$
$$-(\tfrac{16}{5}Q_3^1+\tfrac{9}{5}Q_1^1)\sin\delta\sin(\lambda'-\alpha_1)],$$
$$+p_4^1d_1\,[\{(-\tfrac{4}{3}Q_5^2+\tfrac{5}{6}Q_3^2)\sin(2\lambda'-\alpha_2)+(\tfrac{80}{3}Q_5^0-\tfrac{50}{3}Q_3^0)\sin(-\alpha_2)\}\cos\delta$$
$$-(\tfrac{32}{3}Q_5^1+\tfrac{25}{3}Q_3^1)\sin\delta\sin(\lambda'-\alpha_2)],$$
$$+p_2^2d_1\,[\{-(\tfrac{4}{5}Q_3^3+\tfrac{18}{5}Q_1^1)\sin(3\lambda'-\alpha_2)+(\tfrac{8}{5}Q_3^1-\tfrac{18}{5}Q_1^1)\sin(\lambda'-\alpha_2)\}\cos\delta$$
$$-\tfrac{8}{5}Q_3^2\sin\delta\sin(2\lambda'-\alpha_2)],$$
$$+p_4^2d_1\,[\{(-\tfrac{4}{3}Q_5^3+\tfrac{5}{6}Q_3^3)\sin(3\lambda'-\alpha_2)+(16Q_5^1-25Q_3^1)\sin(\lambda'-\alpha_2)\}\cos\delta$$
$$-(8Q_5^2+10Q_3^2)\sin\delta\sin(2\lambda'-\alpha_2)],$$
$$+p_4^3d_1\,[\{-\tfrac{4}{3}Q_5^4\sin(4\lambda'-\alpha_2)+(8Q_5^2-35Q_3^2)\sin(2\lambda'-\alpha_2)\}\cos\delta$$
$$-(\tfrac{16}{3}Q_5^3+\tfrac{35}{3}Q_3^3)\sin\delta].$$

The values of p_n^σ calculated from the above expressions are given in Table II.

TABLE I.—Velocity Potential $Q_1^1 \sin(\lambda + t - \alpha_1)$.

$$p_2^1 = \frac{1}{6d_0} - \frac{5}{432}\frac{d_1^2 \cos^2\delta}{d_0^3} - \frac{13}{252} \cdot \frac{\cos^2\delta}{d_0^2}\left(d_2 - \frac{d_1^2}{d_0}\right),$$

$$p_1^0 = -\frac{3}{20}\frac{d_1 \cos\delta}{d_0^2},$$

$$p_3^0 = \frac{1}{15}\frac{d_1 \cos\delta}{d_0^2},$$

$$p_1^1 = -\frac{3}{20}\frac{d_1 \sin\delta}{d_0^2},$$

$$p_3^1 = -\frac{2}{45}\frac{d_1 \sin\delta}{d_0^2},$$

$$p_3^2 = -\frac{1}{90}\frac{d_1 \cos\delta}{d_0^2},$$

$$p_2^0 = \frac{1}{72}\frac{d_1^2 \sin\delta\cos\delta}{d_0^3} - \frac{1}{42}\frac{\sin\delta\cos\delta}{d_0^2}\left(d_2 - \frac{d_1^2}{d_0}\right),$$

$$p_4^0 = \frac{2\sin\delta\cos\delta}{35\,d_0^2}\left(d_2 - \frac{d_1^2}{d_0}\right),$$

$$p_4^1 = \frac{1}{140\,d_0^2}(3\cos^2\delta - 1)\left(d_2 - \frac{d_1^2}{d_0}\right),$$

$$p_2^2 = \frac{\sin\delta\cos\delta}{144}\frac{d_1^2}{d_0^3} - \frac{\sin\delta\cos\delta}{84\,d_0^2}\left(d_2 - \frac{d_1^2}{d_0}\right),$$

$$p_4^2 = -\frac{\sin\delta\cos\delta}{210\,d_0^2}\left(d_2 - \frac{d_1^2}{d_0}\right),$$

$$p_4^3 = -\frac{\cos^2\delta}{1680\,d_0^2}\left(d_2 - \frac{d_1^2}{d_0}\right),$$

$$q_2^1 = -\frac{\cos^2\delta}{144} \cdot \frac{d_1^2}{d_0^3} + \frac{\cos^2\delta}{84\,d_0^2}\left(d_2 - \frac{d_1^2}{d_0}\right),$$

$$q_4^1 = -\frac{\cos^2\delta}{280\,d_0^2}\left(d_2 - \frac{d_1^2}{d_0}\right).$$

Table II.—Velocity Potential $Q_2^2 \sin(2\overline{\lambda+t}-\alpha_2)$.

$$p_3^2 = \frac{2}{15 d_0} - \frac{d_1^2}{270 d_0^3} - \frac{2}{45 d_0^2}\left(d_2 - \frac{d_1^2}{d_0}\right),$$

$$p_2^1 = -\frac{16}{63}\frac{d_1 \cos\delta}{d_0^2},$$

$$p_4^1 = \frac{3}{70}\frac{d_1 \cos\delta}{d_0^2},$$

$$p_2^2 = -\frac{8}{63}\frac{d_1 \sin\delta}{d_0^2},$$

$$p_4^2 = -\frac{1}{35}\frac{d_1 \sin\delta}{d_0^2},$$

$$p_4^3 = -\frac{1}{140}\frac{d_1 \cos\delta}{d_0^2},$$

$$p_1^0 = -\frac{8}{35}\frac{\cos^2\delta}{d_0^2}\left(d_2 - \frac{d_1^2}{d_0}\right),$$

$$p_3^0 = -\frac{1}{36}\frac{d_1^2 \cos^2\delta}{d_0^2} + \frac{2}{15}\frac{\cos^2\delta}{d_0^2}\left(d_2 - \frac{d_1^2}{d_0}\right),$$

$$p_5^0 = -\frac{4}{105}\frac{\cos^2\delta}{d_0^2}\left(d_2 - \frac{d_1^2}{d_0}\right),$$

$$p_1^1 = -\frac{8}{35}\frac{\sin\delta\cos\delta}{d_0^2}\left(d_2 - \frac{d_1^2}{d_0}\right),$$

$$p_3^1 = \frac{d_1^2 \sin\delta\cos\delta}{72 d_0^3} - \frac{\sin\delta\cos\delta}{15 d_0^2}\left(d_2 - \frac{d_1^2}{d_0}\right),$$

$$p_5^1 = \frac{16}{525}\frac{\sin\delta\cos\delta}{d_0^2}\left(d_2 - \frac{d_1^2}{d_0}\right),$$

$$p_5^2 = \frac{2}{525}\frac{(3\cos^2\delta - 1)}{d_0^2}\left(d_2 - \frac{d_1^2}{d_0}\right),$$

$$p_3^3 = \frac{d_1^2 \sin\delta\cos\delta}{432 d_0^3} - \frac{\sin\delta\cos\delta}{90 d_0^2}\left(d_2 - \frac{d_1^2}{d_0}\right),$$

$$p_5^3 = -\frac{4}{1575}\frac{\sin\delta\cos\delta}{d_0^2}\left(d_2 - \frac{d_1^2}{d_0}\right),$$

$$p_5^4 = -\frac{\cos^2\delta}{3150 d_0^2}\left(d_2 - \frac{d_1^2}{d_0}\right).$$

As this paper is primarily concerned with the lunar diurnal variation of the earth's magnetism, the numerical values of the coefficients p_n^σ arising from expression (2) owing to the inclination of the magnetic to the geographical axis will not be written down here. This can at once be done, when necessary, from the equations (5), as also the terms in the current potential arising from diurnal and semi-diurnal atmospheric oscillations of degree higher than the type.

§ 21. The main general result of our investigation is the same in form as that of SCHUSTER's more special calculations, viz., that the current function R of electric flow induced under the action of the vertical force $C \cos\theta$ in a shell of air oscillating with a velocity potential $A_m^\tau Q_m^\tau \sin(\tau\,.\,\overline{\lambda+t}-\alpha)$, under the influence of a variable resistivity depending on the zenith distance (ω) of the sun, is

$$(6) \qquad A_m^\tau\Big[\sum_{\sigma=0}^{\infty} p_n^\sigma Q_n^\sigma \sin\{\sigma(\lambda+t)-\alpha\}+\sum_{\sigma=1}^{\infty} q_n^\sigma Q_n^\sigma \sin\{\sigma(\lambda+t)+\alpha\}\Big].$$

In order to obtain the magnetic potential of the variation caused by the flow of air, a factor $-4\pi(n+1)/(2n+1)$ must be inserted before each term Q_n^σ.

We have considered only those terms in the resistivity which depend on $\cos\omega$ and $\cos^2\omega$, though the general theory has been given for any number of terms. If then

$$\kappa = Cae\,(d_0+d_1\cos\omega+d_2\cos^2\omega),$$

we have for the conductivity ρ, to the same degree of approximation,

$$\rho = \frac{1}{Cae\,d_0}\left\{1-\frac{d_1}{d_0}\cos\omega-\frac{1}{d_0}\left(d_2-\frac{d_1^2}{d_0}\right)\cos^2\omega\right\}$$

If we put

$$\frac{1}{Cae\,d_0}=\rho_0, \qquad -\frac{d_1}{d_0}=\frac{\rho_1}{\rho_0}, \qquad -\frac{1}{d_0}\left(d_2-\frac{d_1^2}{d_0}\right)=\frac{\rho_2}{\rho_1},$$

this becomes

$$\rho = \rho_0+\rho_1\cos\omega+\rho_2\cos^2\omega.$$

In SCHUSTER's calculation, the last term was omitted, so that ρ_2 was taken equal to zero, while $\rho_1\cos\delta$ and $\rho_1\sin\delta$ were written $\rho_0\nu$ and $\rho_0\nu'$ respectively. If we make these substitutions in Tables I. and II., it is readily verified that the present results, as far as they go, reduce to those obtained by SCHUSTER. The extra terms depending on $d_2-\dfrac{d_1^2}{d_0}$ give the effect of the term $\cos^2\omega$ in ρ.

§ 22. Finally, a word must be said with regard to the legitimacy of our analysis, considering the fact that if ρ falls to zero, κ', the resistivity, must become infinite. Regarding the matter physically, it is evident that an infinite resistivity is not likely to introduce spurious terms into the current potential, and an examination of the equation (1) for R will show that an actual infinity in κ' would only lead to a zero term in R. But such an infinite term should not occur in the analysis, and it is clear that by altering the constant term in ρ, so that ρ never falls to zero, the above calculations become formally and really legitimate; when we wish to return to the actual case we must appeal to the "law of continuity," and the fact that our mathematics is applied to an ordinary physical problem, to allow us to pass to the limiting value of d_0 in the final result. The latter is expressed as a power series in $1/d_0$, and if d_0 is sufficiently diminished, this series might become non-convergent. But the actual results do not indicate any such behaviour, and are, as we have seen, identical with those obtained by SCHUSTER'S method (in which the conductivity only was considered), so far as the scope of the two calculations is the same.

§ 23. So far the calculations have been kept quite general, in that no relation between the causes of the variable conductivity and of the atmospheric oscillation has been assumed. Thus they may both be caused by the sun, in which case the mathematics is that applicable to the theory of the solar diurnal variations of the earth's magnetism. Without much modification, however, they may equally well be adapted to the case of the lunar diurnal variations. We shall consider it sufficient, for our purpose, to regard the solar and lunar periods as equal at any one time, allowing for the slow cumulative effect of their inequality by introducing a variable phase angle ν into the expression for $\cos\omega$, the quantity on which ρ and κ' depend. Thus

$$\cos\omega = \sin\delta\cos\theta + \cos\delta\sin\theta\cos(\lambda + t' + \nu),$$

where t' is now the local *lunar* time of the standard meridian (measured from upper culmination), and ν measures the lunar phase, increasing from 0 to 2π from one new moon to the next. The velocity potential will be $Q_2^2 \sin(2\overline{\lambda + t'} - \alpha)$. The calculations will be formally the same if we now change the meaning of λ' to $\lambda + t' + \nu$, so that the velocity potential becomes

$$Q_2^2 \sin(2\lambda' - \alpha - 2\nu).$$

Thus by equation (6) the current function obtained is

$$\sum_{\sigma=0}^{\infty} p_n^\sigma Q_n^\sigma \sin\{\sigma\lambda' - \alpha - 2\nu\} + \sum_{\sigma=1}^{\infty} q_n^\sigma Q_n^\sigma \sin(\sigma\lambda' + \alpha + 2\nu)$$
$$= \sum_{\sigma=0}^{\infty} p_n^\sigma Q_n^\sigma \sin\{\sigma(\lambda + t') - \alpha + (\sigma - 2)\nu\} + \sum_{\sigma=1}^{\infty} q_n^\sigma Q_n^\sigma \sin\{\sigma(\lambda + t') + \alpha + (\sigma + 2)\nu\}.$$

The terms on the left of the last line change in phase through an angle $2(\sigma - 2)\pi$ each month, viz., -2π for the diurnal term, zero for the semi-diurnal term, $+2\pi$ for the third component, and $+4\pi$ for the fourth component, as the observations indicated. The terms on the left change phase by $2(\sigma + 2)\pi$ each month, a change so rapid that it would be difficult to detect in the observations, affected as these are by accidental error. The coefficients q_n^σ, moreover, are very small, so that altogether these terms are negligible.

One interesting result of the analysis may be noticed here, viz., that the main lunar term in the magnetic variation, Q_3^2, has a coefficient p_n^σ which does not (to the order of accuracy of our calculations) show any dependence on solar declination. Thus any seasonal change in this term of the magnetic potential cannot be referred to the effect of the varying declination of the sun. This is not quite the case with regard to the main diurnal term in the solar diurnal magnetic variation.

§ 24. We will now consider what are likely values of ρ_1/ρ_0 and ρ_2/ρ_0 to substitute in our formulæ, in order to get a comparison with the observed data. The conductivity should rise to a maximum during the daytime and fall to a minimum about midnight. It cannot actually be less than zero, though it is not so clear that it is better to have the least value of ρ zero than to have it slightly less, in order to make the mean nightly conductivity small in amount. However, we will keep to this condition, and make $\rho_{\text{min.}} = 0$; it is found that the following is a very satisfactory expression for the representation of a function of θ which is large for values of θ up to $\frac{\pi}{2}$, and much smaller, while never negative, from $\theta = \frac{\pi}{2}$ to π:—

$$\rho = \rho_0 (1 + 3\mu + \tfrac{9}{4}\mu^2).*$$

The following table and figure gives the value of $4\rho/\rho_0$ for every 10°. It is seen that the mean of the nine day values is 24·1 times that of the nine night values. The function has a physically false maximum at midnight, but this is of very small amount, and some such feature cannot be avoided with so simple an expression for ρ:—

ω.	0°.	10°.	20°.	30°.	40°.	50°.	60°.	70°.	80°.	90°.
$4\rho/\rho_0$	25·0	24·5	22·2	21·1	18·5	15·4	12·2	9·2	6·4	4·0
ω.	100°.	110°.	120°.	130°.	140°.	150°.	160°.	170°.	180°.	
$4\rho/\rho_0$	2·2	0·9	0·2	0·0	0·1	0·4	0·6	1·0	1·0	

* I might remark here that in working out Part II. of this paper I had not contemplated the possibility of the coefficients of ρ/ρ_0 being greater than unity, as seems to be necessary if the atmospheric conductivity is small and nearly constant at night. The size of these coefficients makes it necessary to carry the calculations some steps further than I have already done, before a sufficient degree of approximation is arrived at. The present work suffices, however, to establish the point with which I am most immediately concerned, viz., that the size of the fourth harmonic in the lunar variation is inexplicable with the form $a + b \cos \omega$ for ρ, while the addition of a term $c \cos^2 \omega$ introduces a fourth harmonic in the theoretical result, which agrees, as to order of magnitude, with the observed quantity. Better observed data are now being obtained, and concurrently I shall proceed to carry the theoretical calculations further, in order to test the exact numerical agreement between theory and observation.—June 11, 1913.

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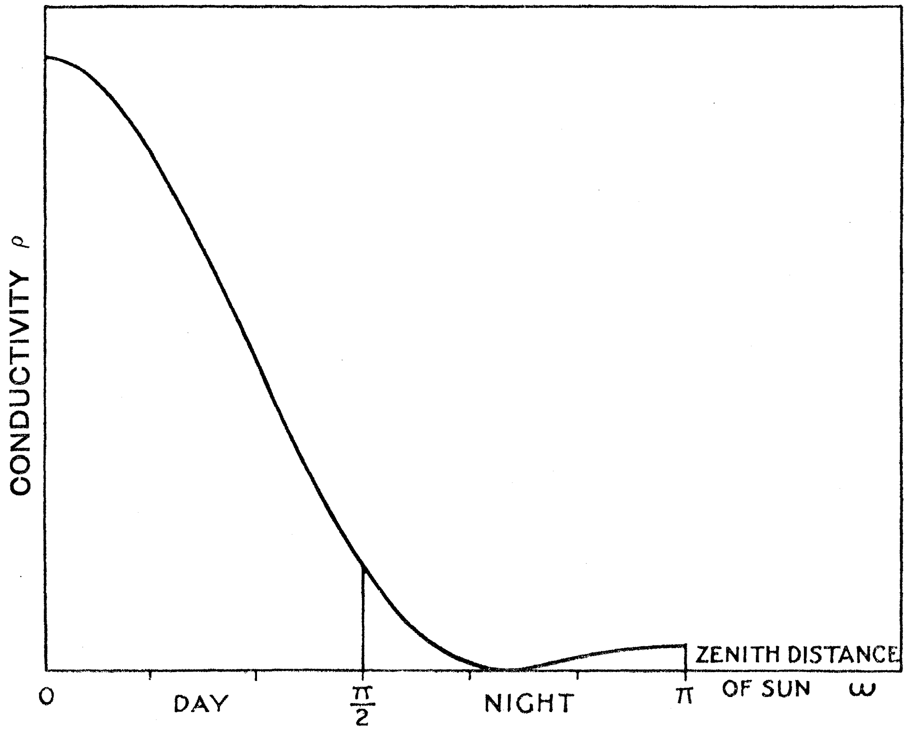

Diagram illustrating the assumed form for the atmospheric conductivity

$$\rho = \rho_0 \left(1 + 3 \cos \omega + \tfrac{9}{4} \cos^2 \omega\right).$$

§ 25. Substituting the values

$$d_1/d_0 = +3, \qquad \left(d_2 - \frac{d_1^2}{d_0}\right)\Big/ d_0 = +\tfrac{9}{4}$$

in the expressions for p_n^σ in Table II. (the table which relates to the lunar variation), we get the following values for $\rho_0 C a e p_n^\sigma$. The terms for which $\sigma = 0$ are omitted, as they merely produce a monthly change in the mean magnetic elements.

$\rho_0 C a e p_n^\sigma$.

σ.	n.				
	1.	2.	3.	4.	5.
1	$\frac{18}{35} \sin\delta \cos\delta$	$\frac{16}{21} \cos\delta$	$\frac{11}{40} \sin\delta \cos\delta$	$-\frac{9}{70} \cos\delta$	$-\frac{12}{175} \sin\delta \cos\delta$
2		$\frac{8}{21} \sin\delta$	$\frac{1}{5}$	$\frac{3}{35} \sin\delta$	$-\frac{3}{350} (3 \cos^2\delta - 1)$
3			$\frac{11}{240} \sin\delta \cos\delta$	$\frac{3}{140} \cos\delta$	$\frac{1}{175} \sin\delta \cos\delta$
4					$\frac{1}{1400} \cos^2\delta$

The following are the values of the corresponding tesseral harmonics:—

$Q_1^1 = \sin\theta$, $\quad Q_2^1 = 3\sin\theta\cos\theta$, $\quad Q_3^1 = \frac{3}{2}\sin\theta\,(5\cos^2\theta-1)$,

$Q_4^1 = \frac{1}{4}\sin\theta\,(70\cos^3\theta-15\cos\theta)$, $\quad Q_5^1 = \frac{5}{8}\sin\theta\,(63\cos^4\theta-84\cos^2\theta+3)$,

$Q_2^2 = 3\sin^2\theta$, $\quad Q_3^2 = 15\sin^2\theta\cos\theta$, $\quad Q_4^2 = \frac{15}{4}\sin^2\theta\,(14\cos^2\theta-1)$,

$Q_5^2 = \frac{105}{2}\sin^2\theta\,(3\cos^3\theta-2\cos\theta)$,

$Q_3^3 = 15\sin^3\theta$, $\quad Q_4^3 = 105\sin^3\theta\cos\theta$, $\quad Q_5^3 = \frac{105}{2}\sin^3\theta\,(9\cos^2\theta-2)$,

$Q_4^4 = 105\sin^4\theta$, $\quad Q_5^4 = 945\sin^4\theta\cos\theta$.

Since all the stations for which we have observational data, in Part III., are tropical, we shall consider the values of X, Y, and Z for such stations only. Hence, in our expression for V, the magnetic potential (which we must now use instead of the current function), all terms containing $\cos^2\theta$ may be neglected, and will be omitted. Thus we get

$$\begin{aligned} V = &\left(\tfrac{459}{280}\cos\delta\,.\,\frac{\sin\theta\cos\theta}{r^2} + \tfrac{57}{1540}\sin\delta\cos\delta\,\frac{\sin\theta}{r^2}\right)\cos(\lambda'+\nu-\alpha), \\ &+\left\{\left(\tfrac{12}{7}+\tfrac{27}{55}\,.\,\overline{3\cos^2\delta-1}\right)\frac{\sin^2\theta\cos\theta}{r^3} + \tfrac{71}{140}\sin\delta\,.\,\frac{\sin^2\theta}{r^3}\right\}\cos(2\lambda'-\alpha), \\ &+\left(\tfrac{5}{4}\,\frac{\sin^3\theta\cos\theta}{r^4}\cos\delta + \tfrac{281}{1540}\sin\delta\cos\delta\,\frac{\sin^3\theta}{r^4}\right)\cos(3\lambda'-\nu-\alpha), \\ &+\tfrac{81}{220}\cos^2\delta\,.\,\frac{\sin^4\theta\cos\theta}{r^5}\cos(4\lambda'-2\nu-\alpha). \end{aligned}$$

In the above expression, the terms depending on $\sin\delta$ represent the main seasonal effect. Since

$$a\mathrm{X} = \frac{dV}{d\theta}, \qquad a\mathrm{Y} = \frac{dV}{\sin\theta\, d\lambda}, \qquad \mathrm{Z} = -\frac{dV}{dr},$$

it is evident that when $\cos\theta$ is put equal to zero after the differentiation, only the terms in V which do not contain $\cos\theta$ will contribute any result to Y and Z. But the above equation shows also that these terms always contain $\sin\delta$, so that at equatorial stations Y and Z change sign in passing from summer to winter. Tables XI. and XIII. corroborate this sufficiently well, especially when it is remembered that the stations are not quite equatorial, and that the obliquity of the magnetic axis also produces a disturbing effect. A further interesting consequence of the above equations is to indicate that at the equator the terms in X which depend on $\sin\delta$, *i.e.*, the seasonal terms in the horizontal force variation, vanish. This agrees with the known fact that at tropical stations the X variation hardly changes throughout the year. Table XII. illustrates this, especially for the most nearly equatorial observatory, Batavia (6° S.).

For comparison with observation we shall write down the values of the ratios of the amplitudes of the first, third, and fourth harmonic components to that of the second; for X we take the mean value of cos δ in our equations, and neglect the seasonal changes; for Y and Z the terms in cos θ and sin δ are taken separately. The values of the amplitudes of the second component in the several cases are also given. It should be remarked that our calculations have not been carried sufficiently far to give the seasonal variation of the fourth component, but it is less important than the term in cos θ, for such stations as Bombay. We thus obtain the following table:—

	C_1/C_2.	C_3/C_2.	C_4/C_2.	C_2.
X	0·61	0·47	0·13	2·61
Y (cos θ)	0·31	0·70	0·27	5·22 cos θ
(sin δ)	0·04	0·55		1·01 sin δ
Z (cos θ)	0·41	0·62	0·22	7·83 cos θ
(sin δ)	0·05	0·49		1·52 sin δ

From SCHUSTER's calculations, taking $\rho/\rho_0 = 1+\cos\omega$, the following table of values of C/C_2, in which the seasonal changes are disregarded, is obtained:—

	C_1/C_2.	C_3/C_2.	C_4/C_2.
X	0·67	0·38	0·002
Y (cos θ)	0·33	0·58	0·003
Z (cos θ)	0·62	0·46	0·0025

Our observational data only allow us to make the roughest possible comparison with these calculations, and the following table is enough to give an idea of what agreement is present. It is got by taking the mean amplitudes at Bombay, Batavia, and Trevandrum (as many as afford data in each case) for the whole year, combining the columns April to September and October to March together by simply averaging the amplitudes regardless of phase.

	C_1/C_2.	C_3/C_2.	C_4/C_2.
X	0·94	0·42	0·28
Y	0·50	0·64	0·23
Z	0·85	1·06	0·47

The size of the fourth harmonic shows that the term $\cos^2\omega$ in ρ/ρ_0 has distinct importance, for without the presence of such a term, as the second of the above tables show, there should be no appreciable fourth harmonic at all. As regards the other harmonics, there is little to chose between the two expressions for ρ/ρ_0, though the more complex one might be made to fit better than the above figures indicate, if the constants of the formula were altered a little. This, however, is not worth while doing till better observational material is to hand.

Part III.—*The Observational Material.*

§ 26. The following are the data which were available for examining the dependence of the lunar magnetic variation upon lunar phase :—

Station and period.	Sub-division of month.	Seasonal division.
	Declination.	
Trevandrum (1854–64)	Four quarters of month	Separate months of year.
Bombay (Chambers) (1846–71) . .	Eight phases	Nov.–Jan., Feb.–April, May–July, Aug.–Oct., April–Sept., Oct.–March.
,, (Moos) (1872–89) . . .	,, ,,	Nov.–Jan.
Batavia (1883–99)	,, ,,	April–Sept., Oct.–March.
	Horizontal Force.	
Bombay (Chambers) (1846–73) . .	Eight phases	As for declination.
,, (Moos) (1872–89) . . .	,, ,,	Nov.–Jan.
,, ,, (1873–79, 1881, 1883–85) . . .	,, ,,	May–July.
Batavia (1883–99)	,, ,,	April–Sept., Oct.–March.
	Vertical Force.	
Bombay (Moos)	Eight phases	As for declination.
Batavia (1883–99)	,, ,,	,, ,,

For purposes of comparison, the Trevandrum results for the separate months of the year have been combined into the four quarters and the two half years (as for Bombay); also the 25 hourly values have been reduced to 24.

The separate tables of the 24 hourly values will not be repeated here, nor the a and b, and C and θ coefficients of the first four harmonic components which have been calculated from those tables. The harmonic formula used has been

$$a_1 \cos t + b_1 \sin t + a_2 \cos 2t + b_2 \sin 2t + a_3 \cos 3t + b_3 \cos 3t + a_4 \cos 4t + b_4 \sin 4t,$$

$$C_1 \sin (t+\theta_1) + C_2 \sin (2t+\theta_2) + \ldots .$$

In the case of all the coefficients a, b, C, the adopted unit is 10^{-7} C.G.S. units of force (the declination results were also reduced in terms of force), and this was reckoned positive towards the North, West, and upwards (H, D, V).

§ 27. The tables of harmonic coefficients showed that they were subject to an accidental error of amount small in itself, but quite a considerable fraction of the whole effect. This is not surprising when the minuteness of the lunar variation is considered. The tables showed some outstanding features, however, in particular the constancy (within reasonable limits) of C_2 and θ_2; C_1, C_3, C_4 are generally smaller and rather more irregular in amount for the eight phases. The phase angles θ_3 showed a fairly regular increase through 2π with the moon's age, while θ_1 showed a less regular decrease of the same amount. No regular change in θ_4 was noticed, partly because C_4 is small and θ_4 therefore not well determined, till the Batavian summer declination results were considered; in this case the fourth harmonic happened to be exceptionally large, and the phase therefore better determined. This clue having once been obtained, the same feature, viz., a monthly increase of 4π in the phase angle θ_4, was verified to be present in most other cases, where C_4 was not too small. The examination of the phase laws followed by the harmonic components was first undertaken by means of vector diagrams, and independently of the theoretical considerations which suggested themselves later, and which are embodied in §§ 12, 23.

The real test of the phase laws suggested by the vector diagrams was made, of course, by correcting the phases by the amount through which the law indicated they had changed from the period of new moon. The corrected values, θ' (where

$$\theta'_1 = \theta_1+\nu,$$

$$\theta'_2 = \theta_2,$$

$$\theta'_3 = \theta_3-\nu,$$

$$\theta'_4 = \theta_4-2\nu,$$

ν being the moon's age, in angular measure, at the particular lunar phase considered) should then all be the same (for the same value of the suffix and different values of ν), apart from accidental error. The Tables III. to X. show that this is the case, generally, as far as we have any right to expect, though, in some instances, the agreement is not very apparent. Even in these cases, however, the mean value of θ' frequently agrees so closely with the mean value of θ'_2 as to show that the phase law is acting, though its manifestation is obscured by large accidental error. This agreement between θ'_1, θ'_2, θ'_3, and θ'_4 is a noticeable feature, of which, as well as of the monthly changes of phase, the theory of the lunar variation gives a satisfactory account (§ 23). On general grounds, too, it is to be expected that if any simple relation exists at all between the phase angles of the four harmonic components, this relation should assume the simplest form (which proves to be equality) at new moon, when the sun and moon are on the same meridian. The equality of the phase angles at new moon points to a single exciting cause (the lunar atmospheric tide being suggested) of the four components.

The regular monthly change in the values of θ_1, θ_3, and θ_4 results in the disappearance of the corresponding harmonic components from the lunar variation, as determined from the mean of a whole number of months. It is found, indeed, that any such component still remaining is of purely accidental character.

As to the amplitude of the various components, this appears to be independent of the lunar phase, the irregularities being accidental. The mean of the amplitudes at the separate phases has therefore been taken as the best value of the true amplitude, except that a correction has been applied to allow for the fact that the instantaneous amplitude is greater than that deduced from the mean of a few days, during which the phase is varying. Thus, if we tabulate a function $c \cos(n\theta + k\nu)$, where θ is the lunar hour angle (one hour $= 15°$) and ν the age of the moon in angular measure, in lunar hours for successive days over an interval of the month ν_1 to ν_2, the mean result may be taken as

$$c(\cos n\theta\, \overline{\cos k\nu} - \sin n\theta\, \overline{\sin k\nu})$$

where $\overline{\cos k\nu}$, $\overline{\sin k\nu}$ are the mean values of these functions over the range ν_1 to ν_2. This equals

$$C\,\frac{\sin \frac{k}{2}(\nu_1-\nu_2)}{\frac{k}{2}(\nu_1-\nu_2)} \cos\left(n\theta + \frac{k}{2}\,\overline{\nu_1+\nu_2}\right),$$

showing that the phase of the mean wave is equal to the true phase at the mean time, but that the amplitude is reduced in the ratio

$$\frac{\sin \frac{k}{2}(\nu_1-\nu_2)}{\frac{k}{2}(\nu_1-\nu_2)}.$$

The corresponding factors to counterbalance this are for Trevandrum, where $\nu_2-\nu_1$ is one-quarter of a month, or $\frac{\pi}{2}$,

$$1{\cdot}11\ (C_1, C_3) \qquad \text{and} \qquad 1{\cdot}57\ (C_4),$$

and at other stations, where $\nu_2 - \nu_1 = \frac{\pi}{4}$,

$$1{\cdot}02\ (C_1, C_3) \qquad \text{and} \qquad 1{\cdot}11\ (C_4).$$

The values of the mean amplitudes, thus corrected, and of the phases of the four components, reduced to new moon, for all the stations, are summarized in Tables XI.–XIII.

The resolved parts of the amplitudes in the direction of the mean phase (where the separate values of θ' depart much from the mean) might have been taken, but this would not have altered the mean amplitude greatly, and seemed hardly worth while in view of the large accidental variations of the determined amplitudes. In Tables XI.–XIII. the values of the mean phases θ' have been characterized by affixes 1, 2, 3, 4, 5, representing (in descending order of merit) the reliability of the mean as judged from the accordance of the separate values of θ'. Only the numbers marked 1 to 3 can be considered at all satisfactory.

As regards the accordance of the results from the same or different stations, the best feature is the extremely good agreement between CHAMBERS' and MOOS'S values for Bombay, for different periods of time and for different instruments.*

* VAN BEMMELEN, in his paper in the 'Met. Zeitschr.,' May, 1912, already referred to, remarks that the two computations do not agree at all, but this must evidently be due to a mistaken reduction of CHAMBERS' results, which he quotes at three times their proper value.

TABLE III.—Trevandrum. Declination West.

Lunar phase.	C_1.	θ'_1.	C_2.	θ_2.	C_3.	θ'_3.	C_4.	θ'_4.
		°		°		°		°
November–January.								
New moon	96	294	204	285	96	306	15	259
First quarter	36	270	162	266	90	252	15	313
Full moon	75	220	162	263	66	296	12	254
Last quarter	45	261	162	277	75	300	9	246
Mean . . .	63	261	172	273	82	288	13	268
February–April.								
New moon	84	5	90	291	45	339	11	43
First quarter	54	3	105	310	60	331	3	6
Full moon	36	−114	102	318	39	367	11	− 9
Third quarter	57	− 29	114	353	51	363	10	86
Mean . . .	58	326	103	313	49	350	9	31

The unit of force in the tables of amplitude is 10^{-7} C.G.S. unit.

TABLE III. (continued).

Lunar phase.	C_1.	θ'_1.	C_2.	θ_2.	C_3.	θ'_3.	C_4.	θ'_4.
		°		°		°		°
May–July.								
New moon	57	115	72	84	60	70	15	62
First quarter	72	72	63	90	30	130	12	272
Full moon	54	43	39	102	36	93	8	27
Third quarter	13	98	42	71	45	96	21	− 37
Mean . . .	49	82	54	87	43	97	14	81
August–October.								
New moon	39	39	36	134	18	149	4	141
First quarter	36	92	42	174	16	214	16	167
Full moon	81	43	63	139	39	160	15	324
Third quarter	33	140	39	184	15	187	17	203
Mean . . .	47	78	45	158	22	177	13	209
April–September.								
New moon	51	73	69	88	48	82	13	59
First quarter	45	50	45	95	24	116	5	− 56
Full moon	66	37	30	106	36	100	6	− 23
Third quarter	12	6	24	67	21	96	10	− 56
Mean . . .	43	42	42	89	32	98	8	341
October–March.								
New moon	60	32	162	282	75	306	6	253
First quarter	12	− 3	135	276	78	296	14	330
Full moon	51	− 141	135	273	48	301	10	294
Third quarter	54	− 57	120	289	51	316	5	228
Mean . . .	44	318	138	280	63	305	9	276

The unit of force in the tables of amplitude is 10^{-7} C.G.S. unit.

TABLE IV.—Bombay (CHAMBERS). Declination West.

Lunar phase.	C_1.	θ'_1.	C_2.	θ_2.	C_3.	θ'_3.	C_4.	θ'_4.
		°		°		°		°
August–October.								
1	73	125	111	112	63	108	18	108
2	19	160	68	150	32	110	19	31
3	68	93	92	150	39	152	23	211
4	41	104	62	131	38	116	12	241
5	33	96	89	124	78	99	20	49
6	53	126	68	127	22	128	6	(82)
7	44	48	83	104	68	89	30	45
8	23	71	93	138	65	117	19	135
Mean . . .	44	103	83	129	51	115	18	117
April–September.								
1	71	97	106	100	55	107	13	− 30
2	45	105	96	117	58	109	18	29
3	55	90	76	119	33	108	14	277
4	52	120	64	91	57	86	11	9
5	33	75	71	108	75	114	17	95
6	60	135	63	124	38	108	20	197
7	17	64	68	90	58	107	8	20
8	31	82	70	120	45	111	14	248
Mean . . .	45	96	66	109	52	106	14	106
October–March.								
1	15	374	83	240	15	257	3	—
2	22	207	62	239	13	433	15	400
3	23	314	82	250	36	239	26	151
4	45	169	62	229	11	148	10	360
5	47	196	36	256	3	—	7	270
6	31	225	54	247	8	297	8	180
7	31	247	66	239	8	209	17	156
8	27	240	62	228	19	176	8	107
Mean . . .	30	246	63	241	14	251	12	232

The unit of force in the tables of amplitude is 10^{-7} C.G.S. unit.

TABLE IV. (continued).

Lunar phase.	C_1.	θ'_1.	C_2.	θ_2.	C_3.	θ'_3.	C_4.	θ'_4.
		°		°		°		°
November–January.								
1	21	256	104	245	29	235	14	168
2	19	304	79	239	13	212	21	424
3	33	327	111	247	62	229	34	209
4	51	184	114	235	36	202	5	(136)
5	79	192	93	255	48	252	32	226
6	66	227	110	229	45	248	12	98
7	85	236	119	245	56	209	33	190
8	60	249	86	237	49	220	7	(266)
Mean . . .	52	247	102	241	42	228	20	219
February–April.								
1	72	392	32	238	19	45	4	—
2	32	283	13	222	42	76	11	32
3	29	265	54	263	18	− 30	13	53
4	43	183	23	318	42	56	18	− 60
5	34	288	32	355	33	17	7	—
6	13	170	22	293	51	48	14	0
7	37	304	13	296	22	55	28	127
8	26	378	30	218	9	40	18	− 26
Mean . . .	36	270	27	275	30	38	17	21
May–July.								
1	71	96	93	98	25	104	34	296
2	77	103	109	106	63	101	17	19
3	58	79	92	109	36	93	17	260
4	93	121	88	93	67	64	24	7
5	43	85	78	102	72	64	28	102
6	57	149	79	113	46	100	29	− 29
7	14	138	66	91	48	96	25	− 55
8	28	18	64	92	51	104	29	240
Mean . . .	55	99	86	101	51	93	25	105

The unit of force in the tables of amplitude is 10^{-7} C.G.S. unit.

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TABLE V.—Bombay (Moos). Declination West.

Lunar phase.	C_1.	θ'_1.	C_2.	θ_2.	C_3.	θ'_3.	C_4.	θ'_4.
		°		°		°		°
November–January.								
1	55	210	143	236	61	233	18	268
2	63	285	58	234	33	178	29	93
3	52	252	145	240	81	217	38	238
4	5	115	96	227	48	205	37	166
5	21	167	129	234	69	233	11	252
6	46	156	125	222	75	215	25	210
7	28	244	80	216	38	230	4	223
8	43	195	96	225	61	204	8	95
Mean . . .	39	203	109	229	58	214	21	193

TABLE VI.—Batavia. Declination West.

Lunar phase.	C_1.	θ'_1.	C_2.	θ_2.	C_3.	θ'_3.	C_4.	θ'_4.
		°		°		°		°
April–September.								
1	38	38	40	104	14	173	12	263
2	59	147	49	167	35	312	20	240
3	20	99	41	115	17	251	10	239
4	43	295	36	13	13	321	14	232
5	1	—	12	6	30	269	28	264
6	16	251	33	119	17	206	20	235
7	18	203	29	255	34	257	21	256
8	11	189	10	270	14	225	28	227
Mean . . .	26	175	31	131	22	264	19	244
October–March.								
1	85	280	238	278	153	291	16	299
2	44	177	172	266	126	290	61	300
3	34	346	175	268	131	283	62	237
4	83	171	130	257	104	289	51	395
5	84	276	237	288	148	296	68	320
6	68	178	154	264	144	283	68	301
7	12	347	181	264	131	282	32	304
8	53	173	148	260	97	284	23	323
Mean . . .	58	243	179	268	129	287	48	310

The unit of force in the tables of amplitude is 10^{-7} C.G.S. unit.

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TABLE VII.—Bombay (CHAMBERS). Horizontal Force.

Lunar phase.	C_1.	θ'_1.	C_2.	θ_2.	C_3.	θ'_3.	C_4.	θ'_4.
		°		°		°		°
				November–January.				
1	124	180	178	188	50	183	28	192
2	68	173	107	179	52	220	23	125
3	103	123	102	152	53	183	14	340
4	155	166	133	178	14	168	3	60
5	78	180	123	172	63	208	29	178
6	161	196	151	173	53	235	29	305
7	136	212	135	195	54	177	47	207
8	191	200	149	177	63	198	38	229
Mean . . .	127	179	135	177	50	198	26	204
				February–April.				
1	68	185	98	164	18	190	15	34
2	96	191	90	165	47	202	52	190
3	78	184	110	156	42	172	8	(135)
4	97	148	93	182	27	285	64	222
5	77	145	48	179	11	(297)	24	178
6	143	181	45	155	21	199	12	124
7	24	(25)	30	(250)	30	344	29	136
8	189	181	70	176	54	225	52	95
Mean . . .	97	174	73	168	31	231	32	140
				May–July.				
1	62	225	50	275	20	158	18	46
2	71	157	88	163	9	103	9	60
3	52	214	83	174	47	135	12	−12
4	33	151	50	101	20	82	18	− 8
5	35	126	16	184	17	164	16	82
6	50	283	60	243	9	25	11	68
7	46	237	16	297	13	66	38	66
8	152	152	60	172	39	269	38	−24
Mean . . .	64	193	53	202	22	125	20	34

The unit of force in the tables of amplitude is 10^{-7} C.G.S. unit.

TABLE VII. (continued).

Lunar phase.	C_1.	θ'_1.	C_2.	θ_2.	C_3.	θ'_3.	C_4.	θ'_4.
		°		°		°		°
August–October.								
1	67	195	24	211	12	309	1	—
2	99	254	76	178	24	127	38	100
3	102	250	41	198	41	283	8	45
4	72	145	92	186	38	207	26	184
5	43	139	48	184	10	246	23	206
6	93	180	36	147	18	326	8	325
7	63	101	27	146	35	33	8	35
8	93	178	72	148	32	2	13	127
Mean . . .	79	180	52	175	26	192	16	146
April–September.								
1	82	211	43	239	9	237	20	54
2	74	195	91	163	8	123	18	141
3	77	234	76	181	8	162	20	– 15
4	72	160	53	158	24	219	9	80
5	35	162	33	197	15	151	17	110
6	33	212	35	197	12	127	7	26
7	23	224	18	278	3	—	22	68
8	136	118	61	171	16	264	11	– 31
Mean . . .	66	190	51	198	12	183	15	54
October–March.								
1	78	173	115	177	34	176	15	171
2	58	201	92	181	51	206	27	139
3	74	140	93	149	50	187	10	138
4	101	153	115	179	8	180	29	208
5	71	147	93	171	37	231	32	205
6	149	185	93	172	39	253	14	287
7	52	186	85	187	30	201	30	183
8	170	193	113	169	52	201	13	166
Mean . . .	94	172	100	173	38	204	21	187

The unit of force in the tables of amplitude is 10^{-7} C.G.S. unit.

TABLE VIII.—Bombay (Moos). Horizontal Force.

Lunar phase.	C_1.	θ'_1.	C_2.	θ_2.	C_3.	θ'_3.	C_4.	θ'_4.
		°		°		°		°
November–January.								
1	129	126	145	170	71	169	43	177
2	162	232	151	177	95	183	11	207
3	164	143	129	179	79	210	36	241
4	86	201	140	190	85	183	44	207
5	101	171	166	180	80	198	14	212
6	161	175	133	151	66	192	69	241
7	112	156	89	152	59	167	17	189
8	113	178	107	172	15	119	38	60
Mean . . .	128	173	132	171	69	178	34	192
May–July.								
1	85	185	48	185	58	306	53	301
2	122	229	22	263	46	6	28	358
3	76	267	67	194	8	261	13	304
4	68	195	28	180	22	51	22	292
5	87	227	72	117	14	210	15	291
6	75	112	40	228	40	192	13	254
7	86	229	18	172	10	186	4	234
8	139	216	88	179	20	165	24	409
Mean . . .	92	207	48	190	27	172	22	305

The unit of force in the tables of amplitude is 10^{-7} C.G.S. unit.

TABLE IX.—Batavia. Horizontal Force.

Lunar phase.	C_1.	θ'_1.	C_2.	θ_2.	C_3.	θ'_3.	C_4.	θ'_4.
		°		°		°		°
April–September.								
1	118	215	99	250	18	251	23	− 34
2	64	126	62	222	36	143	3	—
3	11	72	46	184	18	242	14	93
4	70	169	68	200	24	155	9	22
5	122	221	70	279	29	288	15	− 136
6	11	144	78	197	50	128	1	—
7	43	105	19	173	22	408	14	30
8	55	107	64	213	32	121	32	130
Mean . . .	62	145	63	215	29	217	14	21
October–March.								
1	68	203	86	249	52	230	26	163
2	41	36	52	198	36	166	19	277
3	70	190	115	231	75	205	13	191
4	8	—	80	190	34	149	29	209
5	45	207	74	252	35	204	34	280
6	50	92	68	197	47	315	51	275
7	92	100	98	194	62	200	13	270
8	65	148	95	200	23	125	33	220
Mean . . .	55	122	83	214	45	199	27	236

TABLE X.—Bombay (Moos). Vertical Force (upwards).

Lunar phase.	C_1.	θ'_1.	C_2.	θ_2.	C_3.	θ'_3.	C_4.	θ'_4.
		°		°		°		°
November–January.								
1	23	230	66	272	56	259	27	259
2	10	438	32	289	35	221	19	159
3	22	149	33	246	32	240	7	310
4	12	169	52	259	48	233	20	205
5	32	220	58	262	50	259	18	284
6	31	261	67	276	38	235	10	229
7	16	449	25	279	35	234	20	250
8	12	353	32	273	37	206	16	195
Mean . . .	20	284	46	270	41	236	17	235

The unit of force in the tables of amplitude is 10^{-7} C.G.S. unit.

TABLE X. (continued).—Batavia. Vertical Force.

Lunar phase.	C_1.	θ'_1.	C_2.	θ_2.	C_3.	θ'_3.	C_4.	θ'_4.
		°		°		°		°
April–September.								
1	66	64	157	172	17	200	13	227
2	17	21	95	199	29	284	4	248
3	42	33	86	192	11	176	15	267
4	70	74	50	178	20	345	7	247
5	80	42	94	173	11	421	17	340
6	32	– 20	62	193	54	311	22	293
7	28	7	82	194	21	286	8	264
8	59	– 12	33	181	41	321	9	311
Mean . . .	49	26	82	185	26	293	12	275
October–March.								
1	47	349	36	385	113	12	38	15
2	43	293	44	335	101	0	26	17
3	29	367	17	351	99	6	34	3
4	27	304	26	393	74	– 3	28	9
5	33	427	37	368	88	– 4	33	27
6	94	321	33	293	75	9	59	9
7	47	286	23	327	91	4	20	45
8	14	333	8	306	64	0	42	– 24
Mean . . .	42	335	28	345	88	3	35	13

The unit of force in the tables of amplitude is 10^{-7} C.G.S. unit.

TABLE XI.—Declination West.

	Nov.–Jan.		Feb.–April.		May–July.		Aug.–Oct.		April–Sept.		Oct.–March.	
	C.	θ′.	C.	θ′.	C.	θ′.	C.	θ′.	C.	θ′.	C.	θ′.
						Trevandrum.						
1	70	261^{2}	64	326^{3}	54	82^{2}	52	78^{2}	48	42^{2}	49	318^{3}
2	172	273^{1}	103	313^{1}	54	87^{1}	45	158^{1}	42	89^{1}	138	280^{1}
3	91	288^{2}	54	350^{1}	48	97^{1}	24	177^{2}	36	98^{1}	70	305^{1}
4	20	268^{2}	14	31^{2}	22	81^{5}	20	209^{3}	13	341^{2}	14	276^{2}
						Bombay (CHAMBERS).						
1	53	247^{3}	37	270^{4}	56	99^{2}	45	103^{2}	46	96^{2}	31	246^{3}
2	102	241^{1}	27	275^{3}	86	101^{1}	83	129^{1}	66	109^{1}	63	241^{1}
3	43	228^{2}	31	38^{2}	52	93^{1}	52	115^{1}	53	106^{1}	14	251^{5}
4	22	215^{4}	19	21^{3}	28	105^{5}	20	117^{3}	16	106^{5}	13	232^{5}
						Bombay (Moos).						
1	40	203^{3}										
2	109	229^{1}										
3	59	214^{1}										
4	23	193^{3}										
						Batavia.						
1									27	175^{4}	59	243^{3}
2									31	131^{4}	179	268^{1}
3									23	264^{3}	132	287^{1}
4									21	244^{1}	53	310^{2}

The unit of force in the tables of amplitude is 10^{-7} C.G.S. unit.

TABLE XII.—Horizontal Force.

	Nov.–Jan.		Feb.–April.		May–July.		Aug.–Oct.		April–Sept.		Oct.–March.	
	C.	θ′.	C.	θ′.	C.	θ′.	C.	θ′.	C.	θ′.	C.	θ′.
Bombay (CHAMBERS).												
1	130	179^1	99	174^2	66	193^3	81	180^3	68	190^3	96	172^2
2	135	177^1	73	168^1	53	202^3	52	175^2	51	198^3	100	173^2
3	51	198^2	32	231^4	23	125^4	27	192^4	12	183^3	39	204^3
4	29	204^4	36	140^3	22	34^2	18	146^4	17	54^3	23	187^3
Bombay (Moos).												
1	131	173^3			94	207^3						
2	132	171^1			48	190^3						
3	71	178^2			28	172^3						
4	38	192^3			24	305^3						
Batavia.												
1									64	145^4	56	122^4
2									63	215^2	83	214^2
3									30	217^3	46	199^3
4									16	21^4	30	236^3

TABLE XIII.—Vertical Force.

	Nov.–Jan.		Feb.–April.		May–July.		Aug.–Oct.		April–Sept.		Oct.–March.	
	C.	θ.	C.	θ.	C.	θ.	C.	θ.	C.	θ.	C.	θ.
Bombay (Moos).												
1	20	284^3										
2	46	270^1										
3	42	236^1										
4	19	235^3										
Batavia.												
1									50	26^2	43	335^2
2									82	185^1	28	345^2
3									27	293^3	90	3^1
4									13	275^2	39	13^2

The unit of force in the tables of amplitude is 10^{-7} C.G.S. unit.